MATH 821, SPRING 2012, ASSIGNMENT 3

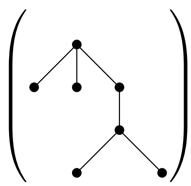
DUE THURSDAY MARCH 14, 2013 IN CLASS

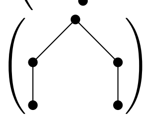
(1) Let V_1, V_2, and V_3 be vector spaces over a field k. Show that $(V_1 \otimes V_2) \otimes V_3$ is isomorphic to $V_1 \otimes (V_2 \otimes V_3)$ using the universal property of tensor products.

(2) Let A be a bialgebra. Prove that the convolution product makes $\mathrm{Hom}(A, A)$ into an algebra.

(3) Let H be a graded, connected, finite-type Hopf algebra. Find an expression for the antipode of H° in terms of the structure functions of H.

(4) Calculate the following things in the Connes-Kreimer Hopf algebra of rooted trees.

  (a) Δ 

  (b) S 

  (c) A characterization of the primitive elements which are homogeneous of degree 3 (note they may be formal sums)

(5) Consider the combinatorial class $\mathcal{C}$ of plane rooted trees. Let $\mathcal{F} = \textsc{Seq}(\mathcal{C})$ which we can view as forests with an order on their component trees.

  (a) Show that $V\mathcal{F}$ can be made into a Hopf algebra where multiplication is concatenation of sequences of trees, and the coproduct on a single tree is the same as in the Connes-Kreimer Hopf algebra of rooted trees except that each $P_c(t)$ inherits an order from the plane structure of t. This is the Foissy or Holtkamp Hopf algebra of plane trees.

  (b) Show that this Hopf algebra is isomorphic to its dual. (If you look it up for hints don't forget to cite your sources)