MATH 923 — PROJECT 3

Report by Wednesday, November 5, 1997

Design a collection of computer programs for solving the minimization problem

$$\min_{x \in \mathbb{R}^n} f(x)$$

using Newton's method. Implement the full step method, as well as line search (the backtracking algorithm described in the book and lecture), and one of the model trust region approaches.

For now write your programs expecting the exact gradients and Hessian matrices. You may want to use the modified Cholesky decomposition. You probably will code some basic stopping criterion; however, if you write everything in MATLAB, you will run the programs interactively, so you can also use your good judgement...

Test your program on a quadratic function f, and on at least one other problem whose exact solution you know. Try to get your programs working as soon as possible. It looks harder than it is.

Problem 1 By minimizing the sum of squares of the "residuals", find a solution of the nonlinear system of equations

$$\begin{aligned} ye^{w+x} &= 2xz \\ w+x+y+z &= y^2 - z^2 \\ wxyz &= x-3 \\ \frac{1}{w+y} + \frac{2}{x+z} &= y. \end{aligned}$$

and of the system

$$\begin{aligned} x_1x_3 - x_2 - x_4^2 &= 1.97 \\ x_3 \sin x_1 - \sqrt{x_4} &= 17.969667 \\ x_2 - 2e^{x_1} &= -0.0201 \\ x_1x_4 + \frac{x_2}{x_3+3} &= 0.041 \end{aligned}$$

Explore various initial guesses, and indicate whether you think the solution is unique. Keep in mind that a solution of the equations is always a minimizer of the sum of squares but not vice versa.

Problem 2 Test your program also on the test functions at the end of the book, Appendix B, pp. 362–363.