MATH 152 — Quiz #1 Solutions

February 6, 1997

Problem 1 Find an antiderivative F of the function f $(F(x) = \int f(x)dx)$. C is a generic constant. (• indicates the correct answer(s)). *Hint:* $\frac{d}{dx}\tan x = 1 + \tan^2 x$.

	$f(x) =$	$F(x) = \int f(x)dx =$	check		$f(x) =$	$F(x) = \int f(x)dx =$	check
a)	$3x + \frac{1}{3x}$	$\frac{3}{2}x^2 + \ln\lvert 3x\rvert + C$	□	c)	$\tan^2 x$	$\tan x - 1 + C$	□
		$\frac{3}{2}x^2 + \frac{1}{3}\ln\lvert x\rvert + C$	•			$\tan x + x + C$	□
		$\frac{3}{2}x^2 + \frac{1}{3}\ln\lvert 3x\rvert + C$	•			$\tan x + C$	□
		$x^3 + \frac{1}{3}\ln\lvert 3x\rvert + C$	□			$\frac{1}{3}\tan^3 x + C$	□
		None of the above	□			None of the above	•
b)	$3x^2 + 3x^{-2}$	$x^3 + 3x^{-1} + C$	□	d)	$\ln(2x)$	$\frac{1}{2x} + C$	□
		$x^3 - 3x^{-1} + C$	•			$x\ln(2x) - x + C$	•
		$x^3 - x^{-3} + C$	□			$2x\ln(2x) - 2x + C$	□
		$x^3 + x^{-3} + C$	□			$x\ln x + x\ln 2 - x + C$	•
		None of the above	□			None of the above	□

Problem 2 What are the values of the following definite integrals. (• indicates the correct answer).

(a)			(b)		
$\int_{-1}^{+1}(x - x^3)dx$ $= 0$ (odd function)	$\frac{1}{2}$	□	$\int_0^{\pi/2}\sin x\cos x\,dx$	$\frac{1}{2}$	•
	$\frac{1}{4}$	□		0	□
	$-\frac{1}{2}$	□		π	□
	$-\frac{1}{4}$	□		1	□
	None of the above	•		None of the above	□

$$(c) \qquad \int_0^{\pi/3}\sin u\,du = [-\cos u]_0^{\pi/3} = -\cos(\pi/3) + \cos(0) = -\frac{1}{2} + 1 = \frac{1}{2}.$$

$$\begin{aligned}(d) \qquad \int_0^2 |x^2 - 1|dx &= \int_0^1 (1 - x^2)dx + \int_1^2 (x^2 - 1)dx \\ &= \left[x - \frac{x^3}{3}\right]_0^1 + \left[\frac{x^3}{3} - x\right]_1^2 = 1 - \frac{1}{3} + \frac{8}{3} - \frac{1}{3} - 2 + 1 = 2.\end{aligned}$$

Problem 3 What is ... (the function g is continuous and differentiable)

$$\frac{d}{dx}\left(\int_0^x e^{t^2}\,dt\right) = e^{x^2}, \qquad \int_0^x \frac{d}{dt}(t^2 - t)\,dt = x^2 - x,$$

$$\frac{d}{dx}\left(\int_1^2 \ln t\,dt\right) = 0, \qquad \frac{d}{dt}\left(\int_t^2 x\,dx\right) = -t,$$

$$\frac{d}{dx}\left(\int_{-x}^{x^2} g(t)\,dt\right) = 2xg(x^2) + g(-x).$$

Problem 4 (20 points) Find the area of the finite region bounded by the curves

$$y = \frac{1}{2}x^2 \qquad \text{and} \qquad y = 2 - x - \frac{1}{2}x^2.$$

Points of intersection: $\frac{1}{2}x^2 = 2 - x - \frac{1}{2}x^2 \Rightarrow x^2 + x - 2 = (x+2)(x-1) = 0.$

$$\begin{aligned} A &= \int_{-2}^{1} (2 - x - \frac{1}{2}x^2 - \frac{1}{2}x^2)dx = \int_{-2}^{1}(2 - x - x^2)dx \\ &= \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{1} = (2 - \frac{1}{2} - \frac{1}{3}) - (-4 - 2 + \frac{8}{3}) = \frac{9}{2}. \end{aligned}$$

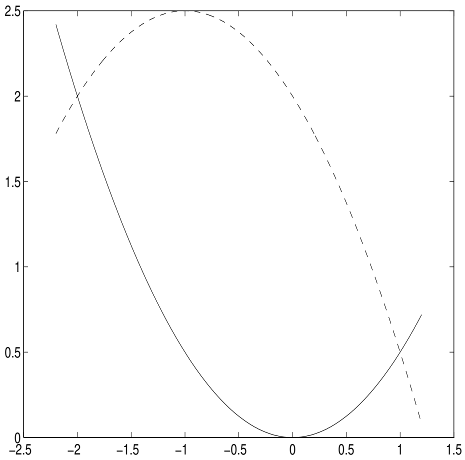

Figure 1: Problem 4

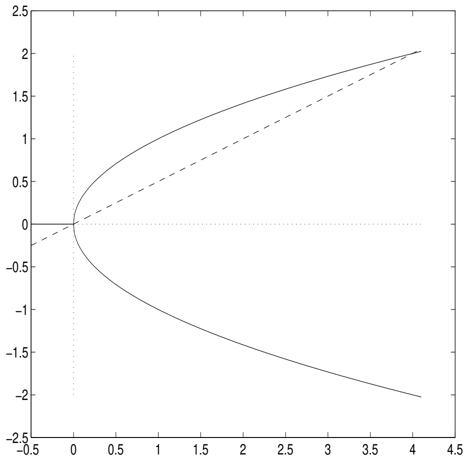

Figure 2: Problem 5

Problem 5 (20 points) Find volume of the solid obtained by rotating the region bounded by the curves

$$y^2 = x \qquad \text{and} \qquad x = 2y$$

about the y-axis. Sketch the region, the solid, and a typical disk or "washer".

Points of intersection: $y^2 = 2y \Rightarrow y = 0,\ x = 0;\ \ y = 2,\ x = 4.$

Solution via cylindrical shells:

$$\begin{aligned} V &= 2\pi \int_0^4 x(\sqrt{x} - \frac{x}{2})dx = 2\pi \int_0^4 (x^{\frac{3}{2}} - \frac{x^2}{2})dx \\ &= 2\pi \left[\frac{2}{5}x^{\frac{5}{2}} - \frac{x^3}{6}\right]_0^4 = 2\pi\left(\frac{2}{5}32 - \frac{32}{3}\right) = 64\pi\left(\frac{2}{5} - \frac{1}{3}\right) = \frac{64}{15}\pi. \end{aligned}$$

Solution via disk slices:

$$\begin{aligned} V &= \pi \int_0^2 \left((2y)^2 - (y^2)^2\right) dy = \pi \int_0^2 (4y^2 - y^4)dy = \\ &= \pi \left[\frac{4}{3}y^3 - \frac{y^5}{5}\right]_0^2 = \pi\left(\frac{32}{3} - \frac{32}{5}\right) = \frac{64}{15}\pi. \end{aligned}$$