MATH 923 — PROJECT 1

Report by Monday, October 01, 2000

Problem 1 Text, page 36, Exercise 1.

Problem 2 Text, page 36, Exercise 7.

Problem 3 Text, page 36, Exercise 9.

Problem 4 Text, page 36, Exercise 10.

Problem 5 Text, page 37, Exercise 11.

Problem 6 Text, page 37, Exercise 12.

Problem 7 (Cubic convergence) We have discussed quadratic convergence.

1. Give a definition of cubic convergence.

2. The iteration function

$$\phi(x) = \frac{1}{2}\left(x + \frac{a}{x}\right) = x - \frac{x^2 - a}{2x}$$

generates an iteration sequence which converges quadratically fast to $\sqrt{a}$. Modify this function to

$$\rho(x) = x - \frac{x^2 - a}{2x} h(x).$$

What are the conditions on $h(\sqrt{a})$, so that the sequence generated by ρ converges to $\sqrt{a}$ at a cubic rate of convergence?

3. Find an h of the form $h(x) = \alpha + \frac{\beta}{x^2}$ satisfying these requirements.

4. Test both formulas (ϕ and ρ) on the computer, and try to conclude which one is more efficient.