MATH 923 — PROJECT 4

Report by Wednesday, November 22, 2000

Write a program implementing the Broyden update for solving the system of nonlinear equations

$$F(x) = 0.$$

You may start with the exact Jacobian at your initial guess, or a finite-difference approximation, or anything else "reasonable".

Test your program on a system whose solution you know (one linear and one nonlinear example).

Problem 1 Use your program to find solutions to the equations in Project 3.

$$\begin{aligned}
ye^{w+x} &= 2xz \\
w + x + y + z &= y^2 - z^2 \\
wxyz &= x - 3 \\
\frac{1}{w+y} + \frac{2}{x+z} &= y.
\end{aligned}$$

and of the system

$$\begin{aligned}
x_1 x_3 - x_2 - x_4^2 &= 1.97 \\
x_3 \sin x_1 - \sqrt{x_4} &= 17.969667 \\
x_2 - 2e^{x_1} &= -0.0201 \\
x_1 x_4 + \frac{x_2}{x_3 + 3} &= 0.041
\end{aligned}$$

Explore various initial guesses. Comments on Broyden vs. minimization techniques of Project 3.

Write a program implementing the penalty method for solving the constrained minimization problem

$$\begin{aligned} &\min f(x) \\ \text{subject to} \quad & g_i(x) \le 0, \quad 1 \le i \le p. \end{aligned}$$

You should be able to use your programs from Project 3 for solving the unconstrained problem

$$\min \left\{ f(x) + c \sum_{i=1}^{p} [\max(0, g_i(x))]^2 \right\}.$$

For problems in two variables include a plot of the admissible region, a contour plot of the function f, and the progress of the sequence x_k generated by the penalty method.

Problem 2

$$\begin{aligned} f(x) &= 2x_1^2 + 2x_1x_2 + x_2^2 - 10x_1 - 10x_2, \\ x_1^2 + x_2^2 &\le 5, \\ 3x_1 + x_2 &\le 6. \end{aligned}$$

Problem 3

$$\begin{aligned} f(x) &= x_1^2 + x_2^2, \\ x_1 + x_2 &\ge 1. \end{aligned}$$

Problem 4

$$\begin{aligned} f(x) &= x_1 + x_2, \\ x_1^2 - x_2 &\le 2. \end{aligned}$$