MATH 929 — PROJECT 2

Report by Tuesday, March 25, 1997

We consider the elliptic partial differential equation

$$(Lu)(x,y) = -\Delta u(x,y) + a_1(x,y)u_x(x,y) + a_2(x,y)u_y(x,y) + c(x,y)u(x,y) = f(x,y), \tag{2.1}$$

on the domain $\Omega = [0,1]^2$, with Dirichlet boundary conditions

$$u = 0 \qquad \text{on } \partial\Omega.$$

We impose a regular grid on Ω with mesh size $h = 1/(n+1)$, which gives us $N = n^2$ unknowns

$$u_{ij} \approx u(ih, jh), \qquad 1 \le i,j \le n.$$

We use the standard five-point centered difference scheme,

$$\begin{aligned} u_x(ih,jh) &\approx \frac{u_{i+1,j} - u_{i-1,j}}{2h}, \\ u_{xx}(ih,jh) &\approx \frac{u_{i+1,j} - 2u_{ij} + u_{i-1,j}}{h^2}, \end{aligned}$$

and similarly for the y-derivatives. After discretization we are left with a linear system

$$Au = f. \tag{2.2}$$

Our linear solver expects the variables u_{ij} in a one-dimensional vector, however, for computing Au it is more efficient to store u as a 2-dimensional array. You will therefore need a routine which converts between these two formats. The adjoint (transpose) of the operator L is given by

$$(L^*u)(x,y) = -\Delta u(x,y) - a_1(x,y)u_x(x,y) - a_2(x,y)u_y(x,y) + c(x,y)u(x,y) \tag{2.3}$$

Write MATLAB codes implementing some or all of the methods we have discussed in class. You can find a description of most of them in the classroom notes, chapter 2, at `http://www.math.sfu.ca/mast/people/faculty/mrt/M929/`. The write-up on TFQMR is not yet completed, but will be available soon.

- **Mandatory:** GMRES, CGNR, CGS, BiCGSTAB
- **Optional:** CGNE, Bi-CG, TFQMR, GMRES(m).

Also, compare performance with and without preconditioner, where the preconditioner is a fast Poisson solver (i.e., an approximate inverse of $-\Delta$). Use the Matlab code `fish2d.m` by C.T. Kelley, given in the software section of the Course Web Page, for the fast Poisson solver. Give comparisons in terms of number of iteration, **and** in terms of total work.

Problem 1 Take

$$a_1(x,y)=1, \qquad a_2(x,y)=20y, \qquad c(x,y)=1,$$

and compute the right hand side such that the discretization of

$$u(x,y)=9xy(1-x)(1-y)\exp(x^{9/2})$$

is an exact solution of (2.2).

Take $n=31$; you may wish to experiment also with larger values of n, keeping in mind that `fish2d.m` is written for values of $n=2^k-1$. **Do not form the matrix** A.

Problem 2 Take

$$a_1(x,y)=100x, \qquad a_2(x,y)=100y, \qquad c(x,y)=-200,$$

and compute the right hand side f such that the discretization of $u\equiv 1$ is an exact solution of (2.2). Use $n=63$. You may also try $n=31$ or $n=127$.