APMA 990 Wavelets — Problem Set 1

Report by Monday, January 28, 2002

Problem 1 Calculate the Fourier series expansions of the following functions, and verify the symmetry properties of the coefficients:

(i) f has period 2, and $f(t) = |t|$ for $|t| < 1$.

(ii) f has period a, and $f(t) = t/a$ for $0 \leq t < a$.

(iii) $f(t) = |\sin t|$.

(iv) $f(t) = \sin^3 t$.

Problem 2 Let c_n be the Fourier coefficients of the periodic function $f : t \to f(t)$. What are the Fourier coefficients of the "delayed" function $g : t \to f(t - t_0)$?

Use your result, and the first problem to find the Fourier coefficients of $f(t) = |\cos t|$.

Problem 3 Let f be periodic with period a, and denote by c_n its Fourier coefficients. Then f is also periodic with period $2a$, and corresponding Fourier coefficients c'_n. What is the relationship between c_n and c'_n ? Verify that the two Fourier series are identical.

Problem 4 Show that for a periodic, twice continuously differentiable function f we have

$$|c_n(f)| \leq \frac{K}{n^2}.$$

Problem 5 We define the convolution of two 2π-periodic functions f and g by

$$(f * g)(s) := h(s) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} f(s - t) g(t) dt.$$

(a) Show that $f * g = g * f$.

(b) Assuming all quantities in question exist, what are the Fourier coefficients of $f * g$ in terms of the Fourier coefficients of f and g ?

(c) If both f and g are in L^2, is the convolution $f * g$ defined ?

Problem 6 Consider two consecutive discrete Fourier transforms:

$$(y_k) \to (Y_n), \qquad (Y_n) \to (z_q).$$

Compute (z_q) as a function of (y_k).

Problem 7 Let (x_k) and (y_k) be two complex N-periodic sequences with

$$x_{N-k} = \bar{x}_k, \qquad y_{N-k} = \bar{y}_k,$$

for all k. Show that the discrete Fourier transforms (X_n) and (Y_n) are real, and that they can be computed with a single transform of order N.

Problem 8 Compute the succesive powers of the Fourier matrix Ω_N.

Problem 9 Calculate the discrete Fourier transform of the vector (x_k) in $\mathbb{C}^N$ with $x_k = k$, for $k = 0, \ldots, N-1$.