

Math 152, [Matlab Diaries](#)

Numerical Integration: Rectangle rule (left (l), and right (r)), Midpoint rule (m), trapezoidal rule (t), Simpson's rule (s).

The experiments show that for "most" functions the numerical integration produces values which have an error proportional to

| Rule | Error proportional to |
|-----------|-----------------------|
| Rectangle | h |
| Midpoint | h^2 |
| trapezoid | h^2 |
| Simpson | h^4 |

where h is the length of the subintervals used.

```
%Commands to compute the definite integral from 0 to 1 of a given
```

```
%function f.
```

```
%Math 152 -- Numerical Integration
```

```
%Define an inline function f, e.g.
```

```
%f=inline('x.^0.5');
```

```
%f=inline('1./(1+x)');
```

```
%f=inline('pi*cos(pi*x/2)');
```

```
%f=inline('1+sin(2*pi*x)');
```

```
%f=inline('(sin(32*pi*x)).^2');
```

```
%Assume interval of integration is [0,1]
```

```
n=4;
```

```
for j=1:7.
```

```
    h=1/n; x=h*[0:n]; xm=h*([0:n-1]+0.5);
```

```
    c=ones(size(x)); c(1)=0.5; c(n+1)=0.5;
```

```
    fx=f(x); fm=f(xm);
```

```
    l(j)=h*sum(fx(1:n)); r(j)=h*sum(fx(2:n+1));
```

```
    m(j)=h*sum(fm); t(j)=h*c*fx'; s(j)=t(j)/3+2*m(j)/3;
```

```
    n=2*n;
```

```
end;
```

```
%END
```

```
----Experiments:
```

```
f =
```

```
    Inline function:
```

```
    f(x) = 2*x
```

```
m152_intnum
```

```
[l;r;m;t;s]'    %Values
```

```
ans =
Columns 1 through 4
0.750000000000000    1.250000000000000    1.000000000000000    1.000000000000000
0.875000000000000    1.125000000000000    1.000000000000000    1.000000000000000
0.937500000000000    1.062500000000000    1.000000000000000    1.000000000000000
0.968750000000000    1.031250000000000    1.000000000000000    1.000000000000000
0.984375000000000    1.015625000000000    1.000000000000000    1.000000000000000
0.992187500000000    1.007812500000000    1.000000000000000    1.000000000000000
0.996093750000000    1.003906250000000    1.000000000000000    1.000000000000000
```

```
Column 5
1.000000000000000
1.000000000000000
1.000000000000000
1.000000000000000
1.000000000000000
1.000000000000000
1.000000000000000
```

```
[l;r;m;t;s]'-1 %Errors
```

```
ans =
Columns 1 through 4
-0.250000000000000    0.250000000000000    0    0
-0.125000000000000    0.125000000000000    0    0
-0.062500000000000    0.062500000000000    0    0
-0.031250000000000    0.031250000000000    0    0
-0.015625000000000    0.015625000000000    0    0
-0.007812500000000    0.007812500000000    0    0
-0.003906250000000    0.003906250000000    0    0
```

```
Column 5
0
0
0
0
0
0
0
```

```
f=inline('1./(1+x)'); %"typical" function
```

```
f,
f =
```

```
Inline function:
f(x) = 1./(1+x)
```

```
m152_intnum
```

```
[l;r;m;t;s]' %Values
```

```
ans =
```

```
Columns 1 through 4
```

```

0.75952380952381    0.63452380952381    0.69121989121989    0.69702380952381
0.72537185037185    0.66287185037185    0.69266055404320    0.69412185037185
0.70901620220753    0.67776620220753    0.69302521433097    0.69339120220753
0.70102070826925    0.68539570826925    0.69311666949756    0.69320820826925
0.69706868888340    0.68925618888340    0.69313955157281    0.69316243888340
0.69510412022811    0.69119787022811    0.69314527323678    0.69315099522811
0.69412469673244    0.69217157173244    0.69314670372438    0.69314813423244
Column 5
0.69315453065453
0.69314765281942
0.69314721028982
0.69314718242146
0.69314718067634
0.69314718056722
0.69314718056040
[1;m;t;s]'-log(2)    %Errors
ans =
0.06637662896386    -0.00192728934005    0.00387662896386    0.00000735009459
0.03222466981191    -0.00048662651674    0.00097466981190    0.00000047225947
0.01586902164758    -0.00012196622897    0.00024402164758    0.00000002972988
0.00787352770930    -0.00003051106239    0.00006102770930    0.00000000186151
0.00392150832346    -0.00000762898713    0.00001525832346    0.00000000011640
0.00195693966816    -0.00000190732317    0.00000381466816    0.00000000000728
0.00097751617250    -0.00000047683557    0.00000095367250    0.00000000000045
f=inline('sin(x).^2')
f =
    Inline function:
    f(x) = sin(x).^2
m152_intnum
[1;m;t;s]'
ans =
0.18892224132172    0.27029030033372    0.27743141860592    0.27267067309112
0.22960627082772    0.27208257185117    0.27386085946982    0.27267533439072
0.25084442133944    0.27252757819089    0.27297171566049    0.27267562401409
0.26168599976517    0.27263863967070    0.27274964692569    0.27267564208903
0.26716231971793    0.27266639317835    0.27269414329820    0.27267564321830
0.26991435644814    0.27267333081417    0.27268026823828    0.27267564328887
0.27129384363116    0.27267506517682    0.27267679952622    0.27267564329329
f=inline('pi*cos(pi*x/2)');
m152_intnum
[1;m;t;s]'
ans =
2.36693068364427    2.01290908559913    1.97423160194555    2.00001659104794
2.18991988462170    2.00321637816795    1.99357034377234    2.00000103336941

```

```

2.09656813139483    2.00080341630993    1.99839336097014    2.00000006453000
2.04868577385238    2.00020081172837    1.99959838864004    2.00000000403226
2.02444329279037    2.00005020028590    1.99989960018420    2.00000000025200
2.01224674653814    2.00001254990610    1.99997490023505    2.00000000001575
2.00612964822212    2.00000313746619    1.99999372507058    2.00000000000099
[1;m;t;s]'-2      %errors
ans =
0.36693068364427    0.01290908559913    -0.02576839805445    0.00001659104794
0.18991988462170    0.00321637816795    -0.00642965622766    0.00000103336941
0.09656813139483    0.00080341630993    -0.00160663902986    0.00000006453000
0.04868577385238    0.00020081172837    -0.00040161135996    0.00000000403226
0.02444329279037    0.00005020028590    -0.00010039981580    0.00000000025200
0.01224674653814    0.00001254990610    -0.00002509976495    0.00000000001575
0.00612964822212    0.00000313746619    -0.00000627492942    0.00000000000099
f=inline('x.^0.5'),
f =
    Inline function:
    f(x) = x.^0.5      %atypical -- errors are "larger than expected
                    %Reason: f is not differentiable on [0,1]
        %Compare error formulas
m152_intnum
[1;m;t;s]'
ans =
0.51828304624275    0.67297739700616    0.64328304624275    0.66307928008502
0.59563022162445    0.66903217213000    0.65813022162445    0.66539818862815
0.63233119687723    0.66753667568066    0.66358119687723    0.66621818274618
0.64993393627894    0.66698268647807    0.66555893627894    0.66650810307836
0.65845831137851    0.66678050321514    0.66627081137851    0.66661060593627
0.66261940729683    0.66670744065623    0.66652565729683    0.66664684620310
0.66466342397653    0.66668121412338    0.66661654897653    0.66665965907443
[1;m;t;s]'-2/3    %Errors
ans =
-0.14838362042392    0.00631073033950    -0.02338362042392    -0.00358738658164
-0.07103644504221    0.00236550546334    -0.00853644504221    -0.00126847803851
-0.03433546978944    0.00087000901399    -0.00308546978944    -0.00044848392049
-0.01673273038772    0.00031601981141    -0.00110773038772    -0.00015856358830
-0.00820835528816    0.00011383654848    -0.00039585528816    -0.00005606073040
-0.00404725936984    0.00004077398956    -0.00014100936984    -0.00001982046357
-0.00200324269014    0.00001454745671    -0.00005011769014    -0.00000700759224
f=inline('(sin(32*pi*x)).^2')
f =
    Inline function:
    f(x) = (sin(32*pi*x)).^2    %Highly oscillatory!!!
m152_intnum

```

```

[1;m;t;s]' %Values: wrong at first, exact once h is sufficiently small
ans =
    0.000000000000000    0.000000000000000    0.000000000000000    0.000000000000000
    0.000000000000000    0.000000000000000    0.000000000000000    0.000000000000000
    0.000000000000000    0.000000000000000    0.000000000000000    0.000000000000000
    0.000000000000000    1.000000000000000    0.000000000000000    0.666666666666667
    0.500000000000000    0.500000000000000    0.500000000000000    0.500000000000000
    0.500000000000000    0.500000000000000    0.500000000000000    0.500000000000000
    0.500000000000000    0.500000000000000    0.500000000000000    0.500000000000000
[1;m;t;s]'-0.5
ans =
   -0.500000000000000   -0.500000000000000   -0.500000000000000   -0.500000000000000
   -0.500000000000000   -0.500000000000000   -0.500000000000000   -0.500000000000000
   -0.500000000000000   -0.500000000000000   -0.500000000000000   -0.500000000000000
   -0.500000000000000    0.500000000000000   -0.500000000000000    0.166666666666667
           0   -0.000000000000000           0   -0.000000000000000
   -0.000000000000000    0.000000000000000   -0.000000000000000    0.000000000000000
   -0.000000000000000           0   -0.000000000000000   -0.000000000000000

```
