5.127 The production-line balancing of an automobile speedometer cup is done as follows. The unbalanced cup is placed on a frictionless shaft at O and is allowed to come to rest. A hole is then punched at A; the cup rotates through an angle θ and again comes to rest. The balancing is completed by punching additional holes at B and C. Knowing that the three holes are equal in size and are at the same distance from the shaft O, determine the required angle α in terms of the angle θ.

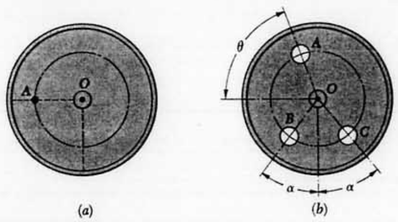


Fig. P5.127

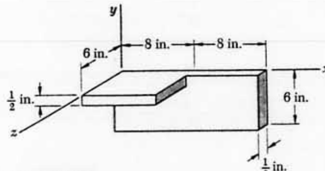


Fig. P5.128

5.128 A portion of one leg of a 6- by 6- by $\frac{1}{2}$-in. angle is cut off. Determine the coordinates of the center of gravity of the remaining portion of the angle.

5.129 Locate the center of gravity of the sheet-metal form shown when $a = 0.2$ m.

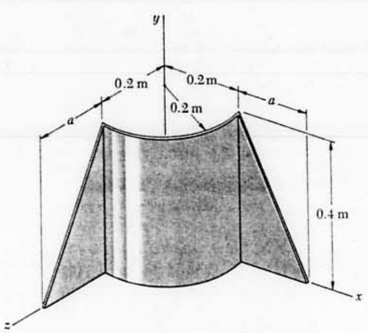


Fig. P5.129 and P5.130

5.130 Determine the distance a so that the center of gravity of the sheet-metal form is located 0.2 m from the y axis.

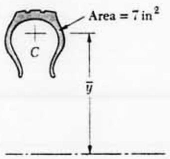


Fig. P5.131

5.131 An automobile tire weighs 22 lb and has a cross-sectional area of 7 in^2. The specific weight of the rubber used is 80 lb/ft^3; determine the location of the centroid of the cross-sectional area.

Analysis of Structures

CHAPTER 6

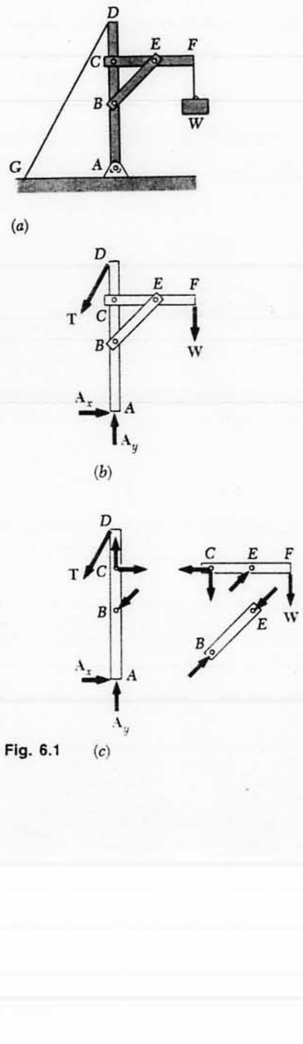


Fig. 6.1

6.1. Internal Forces. Newton's Third Law. The problems considered in the preceding chapters concerned the equilibrium of a single rigid body, and all forces involved were external to the rigid body. We shall now consider problems dealing with the equilibrium of structures made of several connected parts. These problems call not only for the determination of the external forces acting on the structure but also for the determination of the forces which hold together the various parts of the structure. From the point of view of the structure as a whole, these forces are *internal forces.*

Consider, for example, the crane shown in Fig. 6.1*a*, which carries a load W. The crane consists of three beams AD, CF, and BE connected by frictionless pins; it is supported by a pin at A and by a cable DG. The free-body diagram of the crane has been drawn in Fig. 6.1*b*. The external forces are shown in the diagram and include the weight $\mathbf{W}$, the two components $\mathbf{A}_x$ and $\mathbf{A}_y$ of the reaction at A, and the force $\mathbf{T}$ exerted by the cable at D. The internal forces holding the various parts of the crane together do not appear in the diagram. If, however, the crane is dismembered and if a free-body diagram is drawn for each of its component parts, the forces holding the three beams together must also be represented, since these forces are external forces from the point of view of each component part (Fig. 6.1*c*).

It will be noted that the force exerted at B by member BE on member AD has been represented as equal and opposite to the force exerted at the same point by member AD on member

BE; similarly, the force exerted at *E* by *BE* on *CF* is shown equal and opposite to the force exerted by *CF* on *BE*; and the components of the force exerted at *C* by *CF* on *AD* are shown equal and opposite to the components of the force exerted by *AD* on *CF*. This is in conformity with Newton's third law, which states that *the forces of action and reaction between bodies in contact have the same magnitude, same line of action, and opposite sense.* As pointed out in Chap. 1, this law is one of the six fundamental principles of elementary mechanics and is based on experimental evidence. Its application is essential to the solution of problems involving connected bodies.

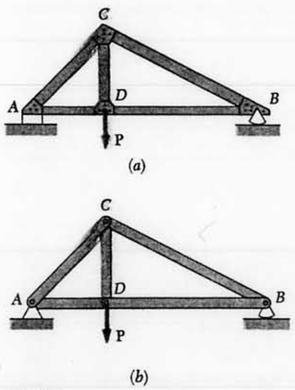


Fig. 6.2

TRUSSES

6.2. Definition of a Truss. The truss is one of the major types of engineering structures. It provides both a practical and an economical solution to many engineering situations, especially in the design of bridges and buildings. A truss consists of straight members connected at joints; a typical truss is shown in Fig. 6.2*a*. Truss members are connected at their extremities only; thus no member is continuous through a joint. In Fig. 6.2*a*, for example, there is no member *AB*; there are instead two distinct members *AD* and *DB*. Actual structures are made of several trusses joined together to form a space framework. Each truss is designed to carry those loads which act in its plane and thus may be treated as a two-dimensional structure.

In general, the members of a truss are slender and can support little lateral load; all loads, therefore, must be applied to the various joints, and not to the members themselves. When a concentrated load is to be applied between two joints, or when a distributed load is to be supported by the truss, as in the case of a bridge truss, a floor system must be provided which, through the use of stringers and floor beams, transmits the load to the joints (Fig. 6.3).

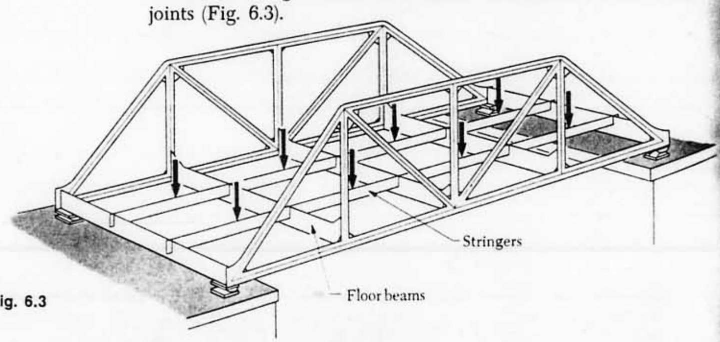


Fig. 6.3

The weights of the members of the truss are also assumed to be applied to the joints, half of the weight of each member being applied to each of the two joints the member connects. Although the members are actually joined together by means of riveted and welded connections, it is customary to assume that the members are pinned together; therefore, the forces acting at each end of a member reduce to a single force and no couple. Thus, the only forces assumed to be applied to a truss member are a single force at each end of the member. Each member may then be treated as a two-force member, and the entire truss may be considered as a group of pins and two-force members (Fig. 6.2*b*). An individual member may be acted upon as shown in either of the two sketches of Fig. 6.4. In the first sketch, the forces tend to pull the member apart, and the member is in tension, while, in the second sketch, the forces tend to compress the member, and the member is in compression. Several typical trusses are shown in Fig. 6.5.

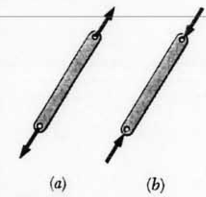


Fig. 6.4

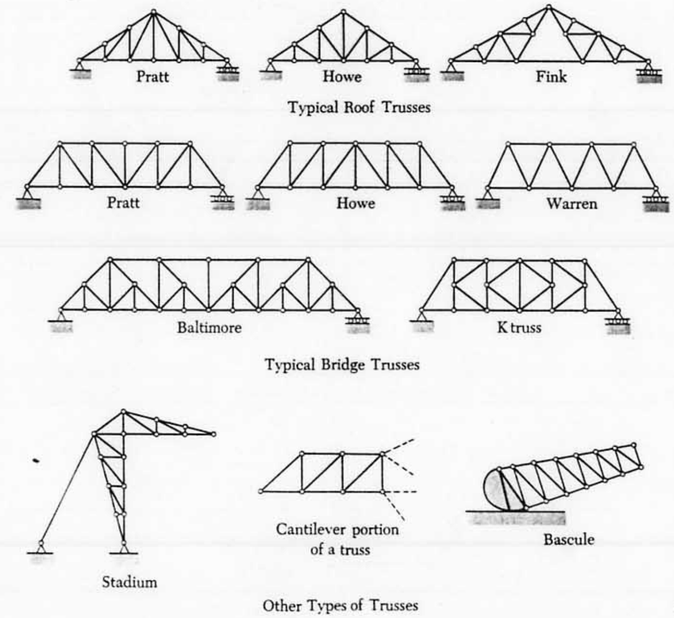


Fig. 6.5

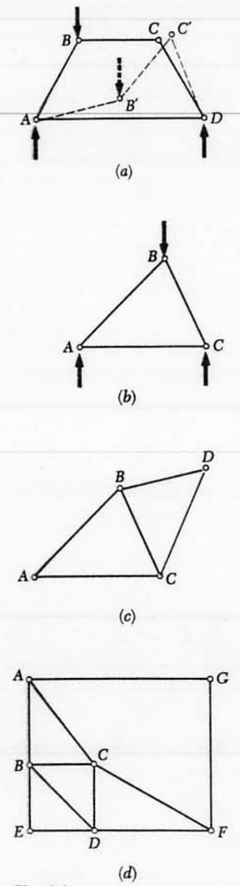


Fig. 6.6

6.3. Simple Trusses. Consider the truss of Fig. 6.6*a*, which is made of four members connected by pins at *A*, *B*, *C*, and *D*. If a load is applied at *B*, the truss will greatly deform and lose completely its original shape. On the other hand, the truss of Fig. 6.6*b*, which is made of three members connected by pins at *A*, *B*, and *C*, will deform only slightly under a load applied at *B*. The only possible deformation for this truss is one involving small changes in the length of its members. The truss of Fig. 6.6*b* is said to be a *rigid truss*, the term rigid being used here to indicate that the truss *will not collapse*.

As shown in Fig. 6.6*c*, a larger rigid truss may be obtained by adding two members *BD* and *CD* to the basic triangular truss of Fig. 6.6*b*. This procedure may be repeated as many times as desired, and the resulting truss will be rigid if, each time we add two new members, we attach them to separate existing joints and connect them at a new joint.† A truss which may be constructed in this manner is called a *simple truss*.

It should be noted that a simple truss is not necessarily made only of triangles. The truss of Fig. 6.6*d*, for example, is a simple truss which was constructed from triangle *ABC* by adding successively the joints *D*, *E*, *F*, and *G*. On the other hand, rigid trusses are not always simple trusses, even when they appear to be made of triangles. The Fink and Baltimore trusses shown in Fig. 6.5, for instance, are not simple trusses, since they cannot be constructed from a single triangle in the manner described above. All the other trusses shown in Fig. 6.5 are simple trusses, as may be easily checked. (For the K truss, start with one of the central triangles.)

Returning to the basic triangular truss of Fig. 6.6*b*, we note that this truss has three members and three joints. The truss of Fig. 6.6*c* has two more members and one more joint, i.e., altogether five members and four joints. Observing that every time two new members are added, the number of joints is increased by one, we find that in a simple truss the total number of members is $m = 2n - 3$, where n is the total number of joints.

6.4. Analysis of Trusses by the Method of Joints. We saw in Sec. 6.2 that a truss may be considered as a group of pins and two-force members. The truss of Fig. 6.2, whose free-body diagram is shown in Fig. 6.7*a*, may thus be dismembered, and a free-body diagram can be drawn for each pin and each member (Fig. 6.7*b*). Each member is acted upon by two forces, one at each end; these forces have the same magnitude, same line of action, and opposite sense (Sec. 4.6). Besides, Newton's third law indicates that the forces of action and reaction between a member and a pin are equal and opposite. Therefore, the forces exerted by a member on the two pins it connects must be directed along that member and be equal and opposite. The common magnitude of the forces exerted by a member on the two pins it connects is commonly referred to as the *force in the member* considered, even though this quantity is actually a scalar. Since the lines of action of all the internal forces in a truss are known, the analysis of a truss reduces to the computation of the forces in its various members and to the determination of whether each of its members is in tension or in compression.

† The three joints must not be in a straight line.

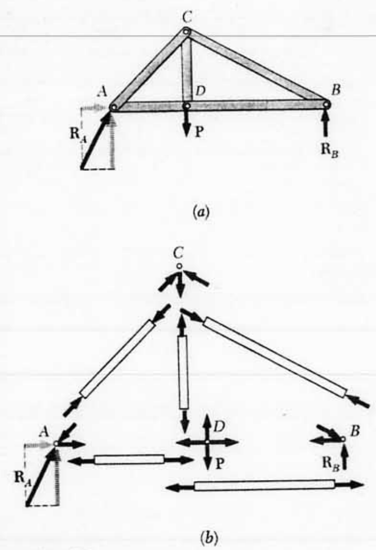


Fig. 6.7

Since the entire truss is in equilibrium, each pin must be in equilibrium. The fact that a pin is in equilibrium may be expressed by drawing its free-body diagram and writing two equilibrium equations (Sec. 2.8). If the truss contains n pins, there will be therefore $2n$ equations available, which may be solved for $2n$ unknowns. In the case of a simple truss, we have $m = 2n - 3$, that is, $2n = m + 3$, and the number of unknowns which may be determined from the free-body diagrams of the pins is thus $m + 3$. This means that the forces in all the members, as well as the two components of the reaction $\mathbf{R}_A$, and the reaction $\mathbf{R}_B$ may be found by considering the free-body diagrams of the pins.

The fact that the entire truss is a rigid body in equilibrium may be used to write three more equations involving the forces shown in the free-body diagram of Fig. 6.7*a*. Since they do not contain any new information, these equations are not independent from the equations associated with the free-body diagrams of the pins. Nevertheless, they may be used to determine immediately the components of the reactions at the supports. The arrangement of pins and members in a simple truss is such that it will then always be possible to find a joint involving only two unknown forces. These forces may be determined by the methods of Sec. 2.10 and their values transferred to the adjacent joints and treated as known quantities at these joints. This procedure may be repeated until all unknown forces have been determined.

As an example, we shall analyze the truss of Fig. 6.7 by considering successively the equilibrium of each pin, starting with a joint at which only two forces are unknown. In the truss considered, all pins are subjected to at least three unknown forces. Therefore, the reactions at the supports must first be determined by considering the entire truss as a free body and using the equations of equilibrium of a rigid body. We find in this way that $\mathbf{R}_A$ is vertical and determine the magnitudes of $\mathbf{R}_A$ and $\mathbf{R}_B$.

The number of unknown forces at joint *A* is thus reduced to two, and these forces may be determined by considering the equilibrium of pin A. The reaction $\mathbf{R}_A$ and the forces $\mathbf{F}_{AC}$ and $\mathbf{F}_{AD}$ exerted on pin A by members AC and AD, respectively, must form a force triangle. First we draw $\mathbf{R}_A$ (Fig. 6.8); noting that $\mathbf{F}_{AC}$ and $\mathbf{F}_{AD}$ are directed along AC and AD, respectively, we complete the triangle and determine the magnitude and sense of $\mathbf{F}_{AC}$ and $\mathbf{F}_{AD}$. The magnitudes F_{AC} and F_{AD} represent the forces in members AC and AD. Since $\mathbf{F}_{AC}$ is directed down and to the left, that is, toward joint A, member AC pushes on pin A and is in compression. On the other hand, since $\mathbf{F}_{AD}$ is directed away from the joint, member AD pulls on pin A and is in tension.

We may now proceed to joint D, where only two forces, $\mathbf{F}_{DC}$ and $\mathbf{F}_{DB}$, are still unknown. The other forces are the load **P**,

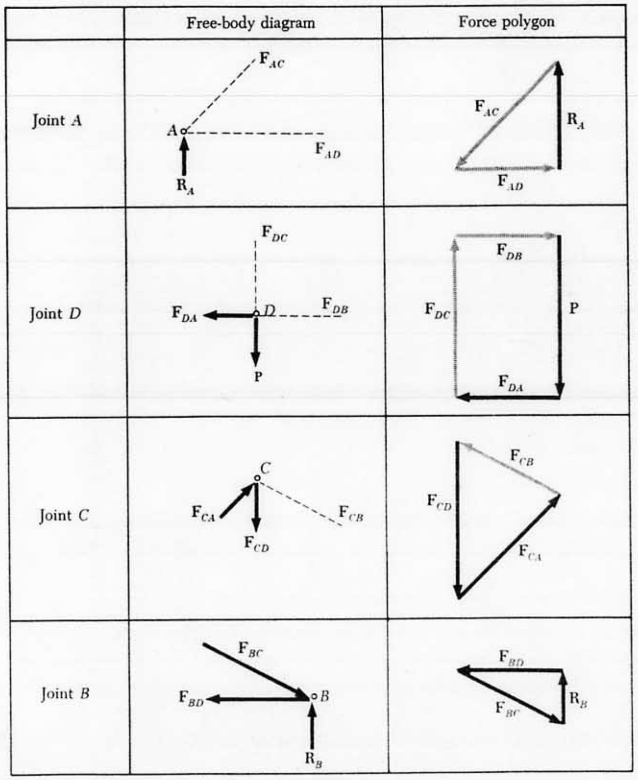


Fig. 6.8

which is given, and the force $\mathbf{F}_{DA}$ exerted on the pin by member AD. As indicated above, this force is equal and opposite to the force $\mathbf{F}_{AD}$ exerted by the same member on pin A. We may draw the force polygon corresponding to joint D, as shown in Fig. 6.8, and determine the forces $\mathbf{F}_{DC}$ and $\mathbf{F}_{DB}$ from that polygon. However, when more than three forces are involved, it is usually more convenient to write the equations of equilibrium $\Sigma F_x = 0$, $\Sigma F_y = 0$ and solve these equations for the two unknown forces. Since both of these forces are found to be directed away from joint D, members DC and DB pull on the pin and are in tension.

Next, joint C is considered; its free-body diagram is shown in Fig. 6.8. It is noted that both $\mathbf{F}_{CD}$ and $\mathbf{F}_{CA}$ are known from the analysis of the preceding joints and that only $\mathbf{F}_{CB}$ is unknown. Since the equilibrium of each pin provides sufficient information to determine two unknowns, a check of our analysis is obtained at this joint. The force triangle is drawn, and the magnitude and sense of $\mathbf{F}_{CB}$ are determined. Since $\mathbf{F}_{CB}$ is directed toward joint C, member CB pushes on pin C and is in compression. The check is obtained by verifying that the force $\mathbf{F}_{CB}$ and member CB are parallel.

At joint B, all the forces are known. Since the corresponding pin is in equilibrium, the force triangle must close and an additional check of the analysis is obtained.

It should be noted that the force polygons shown in Fig. 6.8 are not unique. Each of them could be replaced by an alternate configuration. For example, the force triangle corresponding to

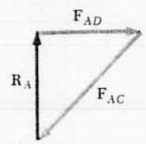


Fig. 6.9

joint A could be drawn as shown in Fig. 6.9. The triangle actually shown in Fig. 6.8 was obtained by drawing the three forces $\mathbf{R}_A$, $\mathbf{F}_{AC}$, and $\mathbf{F}_{AD}$ in tip-to-tail fashion in the order in which their lines of action are encountered when moving clockwise around joint A. The other force polygons in Fig. 6.8, having been drawn in the same way, can be made to fit into a single diagram, as shown in Fig. 6.10. Such a diagram, known as *Maxwell's diagram*, greatly facilitates the *graphical analysis* of truss problems.†

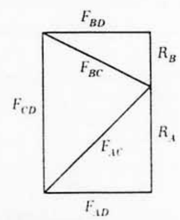


Fig. 6.10

† For a complete discussion of Maxwell's diagrams, see F. P. Beer and E. R. Johnston, "Mechanics for Engineers," sec. 6.7, McGraw-Hill Book Company, 1976.

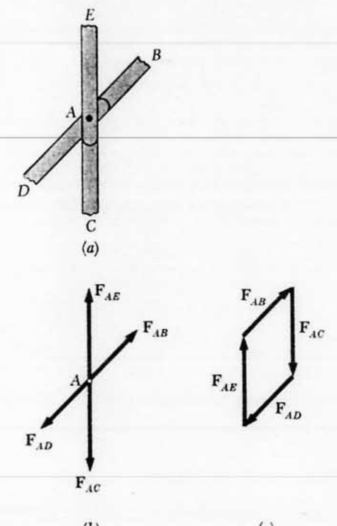


Fig. 6.11

*6.5. Joints under Special Loading Conditions.

Consider the joint shown in Fig. 6.11*a*, which connects four members lying in two intersecting straight lines. The free-body diagram of Fig. 6.11*b* shows that pin A is subjected to two pairs of directly opposite forces. The corresponding force polygon, therefore, must be a parallelogram (Fig. 6.11*c*), and *the forces in opposite members must be equal.*

Consider next the joint shown in Fig. 6.12*a*, which connects three members and supports a load **P**. Two of the members lie in the same line, and the load **P** acts along the third member. The free-body diagram of pin A and the corresponding force polygon will be as shown in Fig. 6.11*b* and *c* with $\mathbf{F}_{AE}$ replaced by the load **P**. Thus, *the forces in the two opposite members must be equal, and the force in the other member must equal P.* A particular case of special interest is shown in Fig. 6.12*b*. Since, in this case, no external load is applied to the joint, we have $P = 0$, and the force in member AC is zero. Member AC is said to be a *zero-force member.*

Consider now a joint connecting two members only. From Sec. 2.8, we know that a particle which is acted upon by two forces will be in equilibrium if the two forces have the same magnitude, same line of action, and opposite sense. In the case of the joint of Fig. 6.13*a*, which connects two members AB and AD lying in the same line, the equilibrium of pin A requires therefore

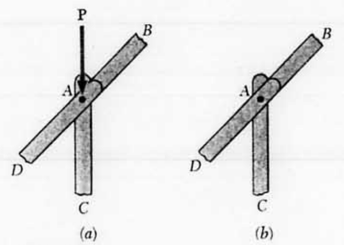


Fig. 6.12

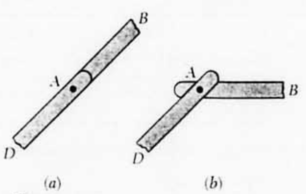


Fig. 6.13

that *the forces in the two members be equal.* In the case of the joint of Fig. 6.13*b*, the equilibrium of pin A is impossible unless the forces in both members are zero. Members connected as shown in Fig. 6.13*b*, therefore, must be *zero-force members.*

Spotting the joints which are under the special loading conditions listed above will expedite the analysis of a truss. Consider, for example, a Howe truss loaded as shown in Fig. 6.14. All the members represented by colored lines will be recognized as zero-force members. Joint C connects three members, two of which lie in the same line, and is not subjected to any external load; member BC is thus a zero-force member. Applying the same reasoning to joint K, we find that member JK is also a zero-force member. But joint J is now in the same situation as joints C and K, and member IJ must be a zero-force member. The examination of joints C, J, and K also shows that the forces in members AC and CE are equal, that the forces in members HJ and JL are equal, and that the forces in members IK and KL are equal. Furthermore, now turning our attention to joint I, where the 20-kN load and member HI are collinear, we note that the force in member HI is 20 kN (tension) and that the forces in members GI and IK are equal. Hence, the forces in members GI, IK, and KL are equal.

Students, however, should be warned against misusing the rules established in this section. For example, it would be wrong to assume that the force in member DE is 25 kN or that the forces in members AB and BD are equal. The conditions discussed above do not apply to joints B and D. The forces in these members and in all remaining members should be found by carrying out the analysis of joints A, B, D, E, F, G, H, and L in the

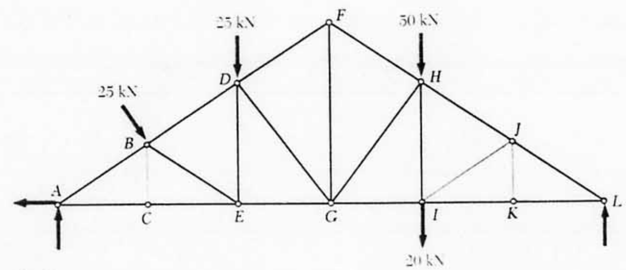


Fig. 6.14

usual manner. Until they have become thoroughly familiar with the conditions of application of the rules established in this section, students would be well advised to draw the free-body diagrams of all pins and to write the corresponding equilibrium equations (or draw the corresponding force polygons), whether or not the joints considered fall into the categories listed above.

A final remark concerning zero-force members: These members are not useless. While they do not carry any load under the particular loading conditions shown, the zero-force members of Fig. 6.14 will probably carry loads if the loading conditions are changed. Besides, even in the case considered, these members are needed to support the weight of the truss and to maintain the truss in the desired shape.

***6.6. Space Trusses.** When several straight members are joined together at their extremities to form a three-dimensional configuration, the structure obtained is called a *space truss.*

We recall from Sec. 6.3 that the most elementary two-dimensional rigid truss consisted of three members joined at their extremities to form the sides of a triangle; by adding two members at a time to this basic configuration, and connecting them at a new joint, it was possible to obtain a larger rigid structure which was defined as a simple truss. Similarly, the most elementary rigid space truss consists of six members joined at their extremities to form the edges of a tetrahedron $ABCD$ (Fig. 6.15*a*). By adding three members at a time to this basic configuration, such as AE, BE, and CE, attaching them at separate existing joints, and connecting them at a new joint, we can obtain a larger rigid structure which is defined as a *simple space truss* (Fig. 6.15*b*).† Observing that the basic tetrahedron has six members and four joints, and that, every time three members are added, the number of joints is increased by one, we conclude that in a simple space truss the total number of members is $m = 3n - 6$, where n is the total number of joints.

If a space truss is to be completely constrained and if the reactions at its supports are to be statically determinate, the supports should consist of a combination of balls, rollers, and balls and sockets which provides six unknown reactions (see Sec. 4.8). These unknown reactions may be readily determined by solving the six equations expressing that the three-dimensional truss is in equilibrium.

Although the members of a space truss are actually joined together by means of riveted or welded connections, it is assumed that each joint consists of a ball-and-socket connection. Thus, no couple will be applied to the members of the truss, and each member may be treated as a two-force member. The conditions of equilibrium for each joint will be expressed by the three equations $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma F_z = 0$. In the case of a simple space truss containing n joints, writing the conditions of equilibrium for each joint will thus yield $3n$ equations. Since $m = 3n - 6$, these equations suffice to determine all unknown forces (forces in m members and six reactions at the supports). However, to avoid solving many simultaneous equations, care should be taken to select joints in such an order that no selected joint will involve more than three unknown forces.

† The four joints must not lie in a plane.

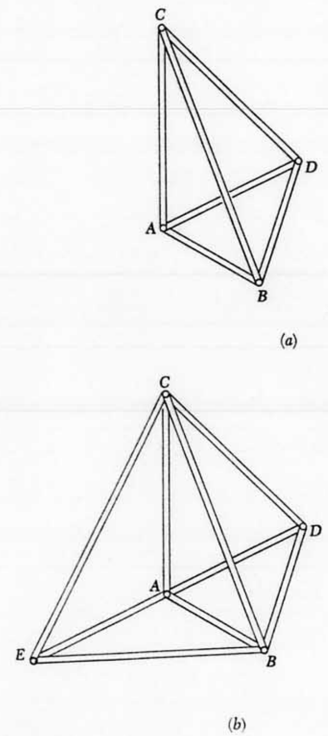
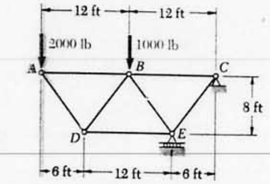
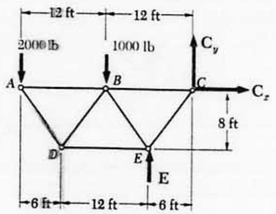


Fig. 6.15

SAMPLE PROBLEM 6.1



Using the method of joints, determine the force in each member of the truss shown.



Solution. A free-body diagram of the entire truss is drawn; external forces acting on this free body consist of the applied loads and the reactions at C and E.

Equilibrium of Entire Truss.

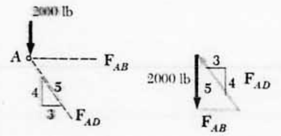
$+\circlearrowleft\Sigma M_C = 0$: $\quad (2000 \text{ lb})(24 \text{ ft}) + (1000 \text{ lb})(12 \text{ ft}) - E(6 \text{ ft}) = 0$

$E = +10{,}000 \text{ lb} \qquad \mathbf{E} = 10{,}000 \text{ lb} \uparrow$

$\xrightarrow{+} \Sigma F_x = 0$: $\qquad \mathbf{C}_x = 0$

$+\uparrow\Sigma F_y = 0$: $\quad -2000 \text{ lb} - 1000 \text{ lb} + 10{,}000 \text{ lb} + C_y = 0$

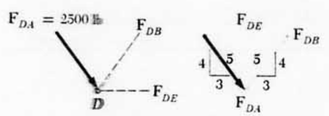
$C_y = -7000 \text{ lb} \qquad \mathbf{C}_y = 7000 \text{ lb} \downarrow$



Joint A. This joint is subjected to only two unknown forces, namely, the forces exerted by members AB and AD. A force triangle is used to determine $\mathbf{F}_{AB}$ and $\mathbf{F}_{AD}$. We note that member AB pulls on the joint and thus is in tension and that member AD pushes on the joint and thus is in compression. The magnitudes of the two forces are obtained from the proportion

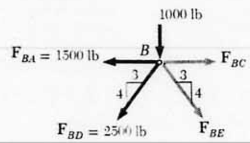
$$\frac{2000 \text{ lb}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

$F_{AB} = 1500 \text{ lb } T$ ◄
$F_{AD} = 2500 \text{ lb } C$ ◄



Joint D. Since the force exerted by member AD has been determined, only two unknown forces are now involved at this joint. Again, a force triangle is used to determine the unknown forces in members DB and DE.

$F_{DB} = F_{DA} \qquad F_{DB} = 2500 \text{ lb } T$ ◄
$F_{DE} = 2(\tfrac{3}{5})F_{DA} \qquad F_{DE} = 3000 \text{ lb } C$ ◄



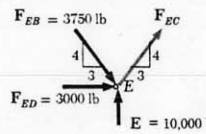
Joint B. Since more than three forces act at this joint, we determine the two unknown forces $\mathbf{F}_{BC}$ and $\mathbf{F}_{BE}$ by solving the equilibrium equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$. We arbitrarily assume that both unknown forces act away from the joint, i.e., that the members are in tension. The positive value obtained for F_{BC} indicates that our assumption was correct; member BC is in tension. The negative value of F_{BE} indicates that our assumption was wrong; member BE is in compression.

$+\uparrow\Sigma F_y = 0$: $\quad -1000 - \tfrac{4}{5}(2500) - \tfrac{4}{5}F_{BE} = 0$

$F_{BE} = -3750 \text{ lb} \qquad F_{BE} = 3750 \text{ lb } C$ ◄

$\xrightarrow{+} \Sigma F_x = 0$: $\quad F_{BC} - 1500 - \tfrac{3}{5}(2500) - \tfrac{3}{5}(3750) = 0$

$F_{BC} = +5250 \text{ lb} \qquad F_{BC} = 5250 \text{ lb } T$ ◄



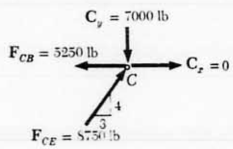
Joint E. The unknown force $\mathbf{F}_{EC}$ is assumed to act away from the joint. Summing x components, we write

$\xrightarrow{+} \Sigma F_x = 0$: $\quad \tfrac{3}{5}F_{EC} + 3000 + \tfrac{3}{5}(3750) = 0$

$F_{EC} = -8750 \text{ lb} \qquad F_{EC} = 8750 \text{ lb } C$ ◄

Summing y components, we obtain a check of our computations:

$$+\uparrow\Sigma F_y = 10{,}000 - \tfrac{4}{5}(3750) - \tfrac{4}{5}(8750)$$
$$= 10{,}000 - 3000 - 7000 = 0 \qquad \text{(checks)}$$



Joint C. Using the computed values of $\mathbf{F}_{CB}$ and $\mathbf{F}_{CE}$, we may determine the reactions $\mathbf{C}_x$ and $\mathbf{C}_y$ by considering the equilibrium of this joint. Since these reactions have already been determined from the equilibrium of the entire truss, we will obtain two checks of our computations. We may also merely use the computed values of all forces acting on the joint (forces in members and reactions) and check that the joint is in equilibrium:

$\xrightarrow{+} \Sigma F_x = -5250 + \tfrac{3}{5}(8750) = -5250 + 5250 = 0 \qquad$ (checks)
$+\uparrow\Sigma F_y = -7000 + \tfrac{4}{5}(8750) = -7000 + 7000 = 0 \qquad$ (checks)

PROBLEMS

6.1 through 6.12 Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

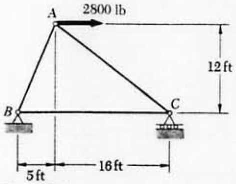


Fig. P6.1

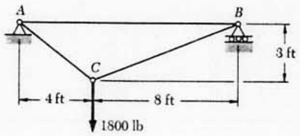


Fig. P6.2

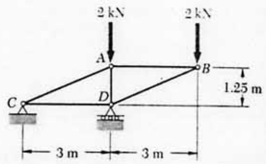


Fig. P6.3

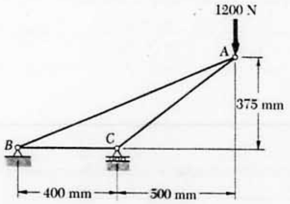


Fig. P6.4

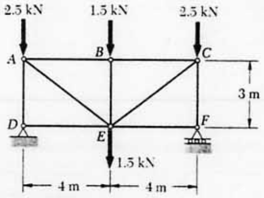


Fig. P6.5

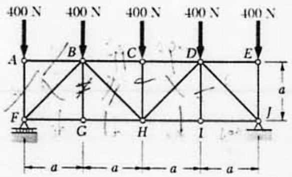


Fig. P6.6

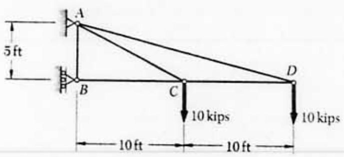


Fig. P6.7

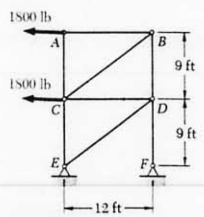


Fig. P6.8

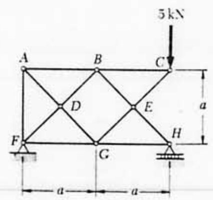


Fig. P6.9

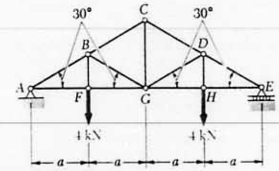


Fig. P6.10

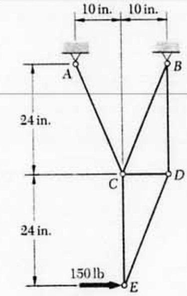


Fig. P6.11

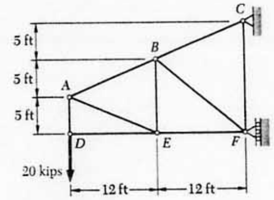


Fig. P6.12

6.13 Determine whether the trusses given in Probs. 6.7, 6.9, 6.14, 6.15, and 6.16 are simple trusses.

6.14 through 6.16 Determine the zero-force members in the truss shown for the given loading.

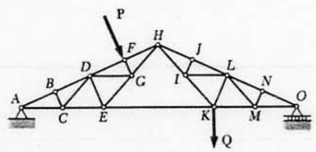


Fig. P6.14

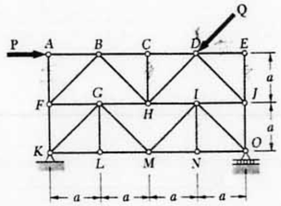


Fig. P6.15

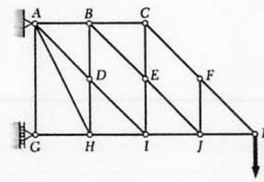


Fig. P6.16

∗6.17 Twelve members, each of length L, are connected to form a regular octahedron. Determine the force in each member if two vertical loads are applied as shown.

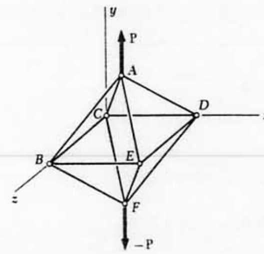


Fig. P6.17