A) BUSINESS

- paper assignment due: MON 08 SEPT
- online " " : THU 11 SEPT

B) LAST DAY

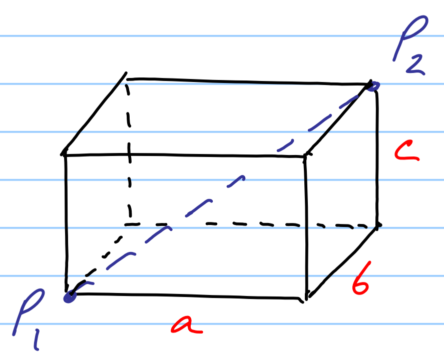
- coordinates of a point in 3 dimensions: $P = (x, y, z)$
- sets of points & equations of surfaces

$$S_1 = \{(x,y,z) \mid x=0,\ y=0\} \rightarrow$$

$$S_2 = \{(x,y) \mid x+y=1\} \rightarrow$$

$$S_3 = \{(x,y,z) \mid x+y=1\} \rightarrow$$

- length of diagonal of a box



$$|P_1P_2| = \sqrt{a^2+b^2+c^2}$$

by repetitive use of Pythagorean Formula.

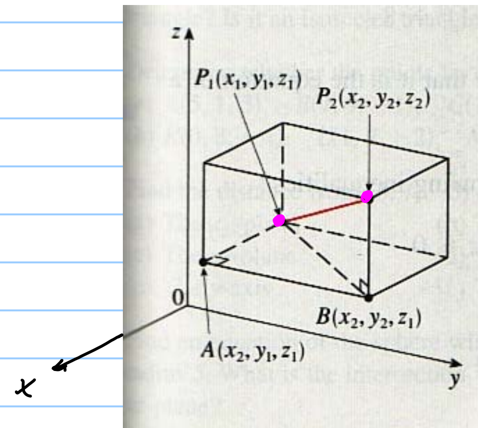
(WE's notes)

c) DISTANCE FORMULA FOR 2 POINTS

- 2 points in $\mathbb{R}^3$ define the opposite corners of a rectangular box

$P_1 = (x_1, y_1, z_1)$

$P_2 = (x_2, y_2, z_2)$



URE 9

side in x-direction $= |P_1A|$

$= | \qquad |$

and $|AB| = | \qquad |$

$|BP_2| = | \qquad |$

$$|P_1P_2| = \sqrt{\qquad^2 + \qquad^2 + \qquad^2}$$

$$= \sqrt{\qquad^2 + \qquad^2 + \qquad^2}$$

p767

3) WHAT IS THE SET OF POINTS WHOSE DISTANCE FROM $C = (h, k, \ell)$ is r $(r > 0)$?

(a) $S = \{ (x,y,z) \mid \sqrt{(\quad)^2 + (\quad)^2 + (\quad)^2} = \quad \}$

OR $S = \{ (x,y,z) \mid \underline{(x-h)^2 + (y-k)^2 + (z-\ell)^2 = r^2} \}$

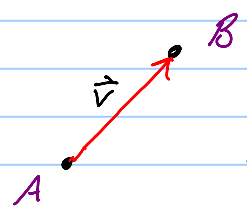
(b) a SPHERE of radius r centred at the point $C = (h, k, \ell)$

equation for a sphere in $\mathbb{R}^3$ → spherical surface

• read examples 5 & 6 (p768) "the CANTALOUPE" via inequality

E) VECTORS (12.2)

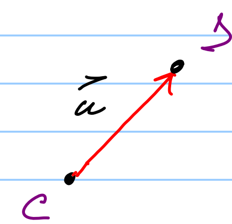
- a vector is a geometrical quantity that has both ________ & ________. It is represented by an arrow (or directed line segment) in space



denoted by $\vec{v} =$

- although represented as an arrow from the initial point A to the terminal point B, only the relative displacement is important

as we say the vectors $\vec{v}$ & $\vec{u}$ are equal.

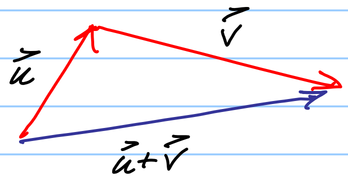


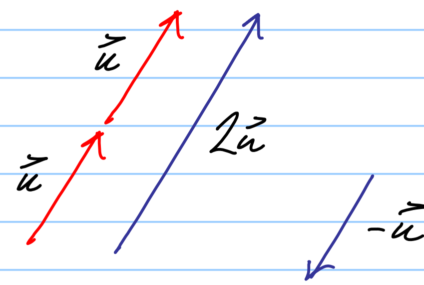
- vectors are NOT associated with any fixed point or location in space

( )

F) ARITHMETIC NATURE OF VECTORS (p 770-772)

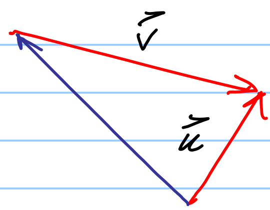
- vectors can be added & scalar multiplied

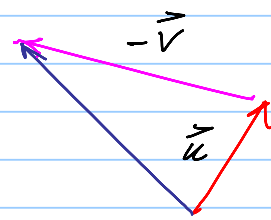




scalar multiply: same orientation, magnitude is multiplied by a real number

( )





=

- magnitude of a vector =

Properties of Vectors If **a**, **b**, and **c** are vectors in V_n and c and d are scalars, then

1. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$
2. $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$
3. $\mathbf{a} + \mathbf{0} = \mathbf{a}$
4. $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$
5. $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$
6. $(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}$
7. $(cd)\mathbf{a} = c(d\mathbf{a})$
8. $1\mathbf{a} = \mathbf{a}$

(p 774)

G) ALGEBRAIC NATURE OF VECTORS (p772-

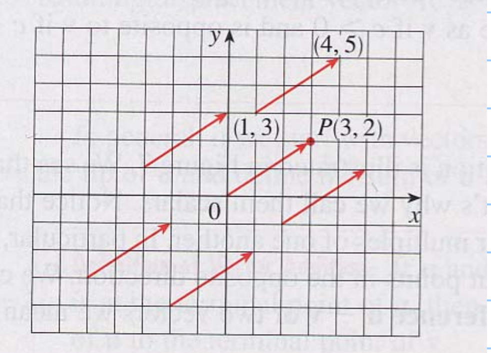


FIGURE 12
Representations of the vector $\mathbf{a} = \langle 3, 2 \rangle$

- embed vector in coordinate system
- place initial point at
- denote vector by coordinates of arrowhead. $\vec{a} =$
- works in $\mathbb{R}^2$ & $\mathbb{R}^3$

WARNING: do not confuse POINTS & VECTORS

- both can be uniquely identified by coordinates, but are distinct concepts

- for a 3D vector having initial point $A = (x_1, y_1, z_1)$ & arrow point $B = (x_2, y_2, z_2)$

$$\vec{AB} = \langle \quad , \quad , \quad \rangle$$

(p773)

- magnitude formula for $\vec{u} = \langle a, b, c \rangle \in \mathbb{R}^3$

$$|\vec{u}| = \sqrt{\quad}$$

multiplicity of notation

$|a| =$

$|P_1P_2| =$

$|\vec{a}| =$

If $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$, then

$$\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2 \rangle \qquad \mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2 \rangle$$

$$c\mathbf{a} = \langle ca_1, ca_2 \rangle$$

Similarly, for three-dimensional vectors,

$$\langle a_1, a_2, a_3 \rangle + \langle b_1, b_2, b_3 \rangle = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$

$$\langle a_1, a_2, a_3 \rangle - \langle b_1, b_2, b_3 \rangle = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$$

$$c\langle a_1, a_2, a_3 \rangle = \langle ca_1, ca_2, ca_3 \rangle$$

- special vectors ($\mathbb{R}^3$)

$\vec{0}$ = zero vector $= \langle \ , \ , \ \rangle$

unit basis vectors: $\hat{i} = \langle \ , \ , \ \rangle$

$\hat{j} = \langle \ , \ , \ \rangle$

$|\hat{i}| = 1$, etc

$\hat{k} = \langle \ , \ , \ \rangle$

$\bar{a} = \langle x_1, y_1, z_1 \rangle = \underbrace{\quad \hat{i} + \quad \hat{j} + \quad \hat{k}}_{\text{scalar mult.}}$

H) APPLICATIONS

- quantifying geometry (magnitude + direction)
- familiar vectors of physics

VELOCITY

$\overrightarrow{\text{ground speed}} = \overrightarrow{\text{wind speed}} + \overrightarrow{\text{air speed}}$ (#30)

FORCE

i) net force on an object is the sum of all applied forces (vectors) (#28)

ii) the forces acting on a stationary object sum (as vectors) to zero (vector) (ex 7, p776 + #34)