Systems of Differential Equations. Another classification of differential equations depends on the number of unknown functions that are involved. If there is a single function to be determined, then one equation is sufficient. However, if there are two or more unknown functions, then a system of equations is required. For example, the Lotka–Volterra, or predator–prey, equations are important in ecological modeling. They have the form

2 couples ODEs

solve for $x(t)$ & $y(t)$

$$\begin{aligned} dx/dt &= ax - \alpha xy \\ dy/dt &= -cy + \gamma xy, \end{aligned} \tag{4}$$

where $x(t)$ and $y(t)$ are the respective populations of the prey and predator species. The constants a, α, c, and γ are based on empirical observations and depend on the particular species being studied. Systems of equations are discussed in Chapters 7 and 9; in particular, the Lotka–Volterra equations are examined in Section 9.5. It is not unusual in some areas of application to encounter systems containing a large number of equations.

Order. The **order** of a differential equation is the order of the highest derivative that appears in the equation. The equations in the preceding sections are all first order equations, while Eq. (1) is a second order equation. Equations (2) and (3) are second order partial differential equations. More generally, the equation

$$F[t, u(t), u'(t), \ldots, u^{(n)}(t)] = 0 \tag{5}$$

is an ordinary differential equation of the nth order. Equation (5) expresses a relation between the independent variable t and the values of the function u and its first n derivatives $u', u'', \ldots, u^{(n)}$. It is convenient and customary in differential equations to write y for $u(t)$, with $y', y'', \ldots, y^{(n)}$ standing for $u'(t), u''(t), \ldots, u^{(n)}(t)$. Thus Eq. (5) is written as

$$F(t, y, y', \ldots, y^{(n)}) = 0. \tag{6}$$

For example,

$$y''' + 2e^t y'' + yy' = t^4 \tag{7}$$

is a third order differential equation for $y = u(t)$. Occasionally, other letters will be used instead of t and y for the independent and dependent variables; the meaning ... from the context.