

Homework #1 • MATH 314 • Linear Algebra Review

- please respect page limits. Have each problem start at the top of a new page.
- submit your write-up into the Math314 box by 4pm, Friday 18 January.
- remember that webct is an open forum for discussion.
- please acknowledge collaborations & assistance from colleagues.

- read the *Guideline* for assignments as posted on the class webpage.
- especially for this assignment, written descriptions of your calculational steps are essential

A) Linear Combinations (2 pages max, 10pts) Given the 3-component vectors

$$\vec{w}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} ; \quad \vec{w}_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} ; \quad \vec{w}_3 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} ; \quad \vec{b} = \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix} ; \quad \vec{f} = \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix}$$

express the vector \vec{b} as a linear combination of the vector set $\mathcal{S} = \{\vec{w}_j\}_{j=1 \rightarrow 3}$. Give as many one sentence reasons as you can that explains why your representation is unique.

Finally, explain why the vector set $\mathcal{S}_f = \{\vec{w}_1, \vec{w}_2, \vec{f}\}$ is not linearly independent.

B) A Real Symmetric Matrix (3 pages max, 15pts) Given the 3×3 \mathbb{R} -valued matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

use Gaussian elimination (choose an efficient strategy) to solve the system of linear equations

$$\mathbf{A} \vec{x} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} = \vec{b}$$

for the 3-component vector \vec{x} .

Give all of the eigenvalues and eigenvectors of the matrix \mathbf{A} . You need only present the detailed derivation for one of the eigenvectors.

Redo the above linear solve for \vec{x} by invoking theorems that follow from the symmetry of the matrix \mathbf{A} .

C) Complex-Valued Linear Algebra (2 pages max, 10pts) Find all of the eigenvalues and eigenvectors for the \mathbb{C} -valued matrix

$$\mathbf{B} = \begin{bmatrix} 1 & 2i \\ -2i & 1 \end{bmatrix}$$

and give an efficient solution to the linear system

$$\mathbf{B} \vec{y} = \begin{bmatrix} 1 & 2i \\ -2i & 1 \end{bmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} .$$