## Math 314 • Boundary Value Problems • Spring 2008

## Elementary Partial Differential Equations: Theory, Computing \& Graphics

What we perceive of the world around us are variations of physical effects (like heat, sound \& light) over space and time. Partial differential equations (PDEs) are the mathematical concept for describing this sensory landscape in terms of continuous functions. This year's course contains the core of the traditional boundary value problems curriculum, but will also introduce the computer graphics and numerical computational tools associated with the analysis of PDEs and their solutions.

Central to the theory of linear PDEs are the Fourier series and Fourier transform. The numerical implementation of the Fourier series, the fast Fourier transform (FFT), is one of the most important numerical algorithms in scientific computing. The "three stars" of elementary PDEs: the potential, heat and wave equations will be introduced through their Fourier solutions. The generalization of these to higher dimensions will naturally lead to the "special" functions, such as the Bessel function and spherical harmonics.

The computational tools will be developed from numerical routines based upon the linear algebra of matrices and vectors. The numerical computing and graphics will be performed through the modification of downloaded Matlab scripts, and the occasional Maple worksheet.

Textbook: Applied Partial Differential Equations, DuChateau \& Zachmann, Dover.
Prerequisites: Math 252 or 254 \& Math 310; or instructor's permission.
Further information \& updates: www.math.sfu.ca/~muraki


The image on the left shows the primary (lower) and secondary rainbows, with the so-called Alexander's dark band in between. The plot on the right shows backreflected light intensity (logscale) versus viewing angle of a "separation of variables" rainbow computed using Bessel functions. This plot, based upon a solution of the Helmholtz PDE, predicts (noisy) intensity peaks very near the $42^{\circ}$ and $52^{\circ}$ angles (red lines) as observed for the familiar rainbows.

