Homework #2 • MATH 322 • Mappings & Limits I

- please respect page limits. Have each problem start at the top of a new page.
- read the *Guideline* for assignments as posted on the class webpage.
- submit your write-up into your Section's box by noon, Friday 22 September.
- remember that webct is an open forum for discussion.
- please acknowledge collaborations & assistance from colleagues.

A) Two Mappings (3 pages max, 10pts)

i) Consider Problem #7 of page 42. In your discussion, provide clear analyses which explain where the corner points and coordinate lines (horizontal & vertical) are mapped. Include a handdrawn figure to illustrate key features of the map.

ii) A linear map with the form $w = f(z) = Az + B$ takes the unit square with corners at $\{0, 1, 1+i, i\}$ to another square such that:

- the area is four times larger;
- the sides are rotated $\pi/6$ counter-clockwise (positive sense);
- the new center is located at $6 + 8i$.

Explain how to find a set of complex-valued constants, A, B, that performs this map. Be clear about your method, as there is more than one such set. How many such sets of constants are there?

B) Limit Proofs (3 pages max, 10pts)

i) Use an ϵ-δ argument to prove the limit of Problem #1c of page 53. For this problem, you should derive an explicit formula for the $\delta(\epsilon)$.

ii) Invoke all appropriate theorems involving the arithmetic of limits to substantiate the limit of Problem #3b of page 53.

C) No-Limit Proof (3 pages max, 10pts)

i) Present the argument as suggested in Problem #5 of page 53 to prove the non-existence of the limit.

ii) Re-express the complex-valued limit into the equivalent real-valued form as used in Theorem 1 of Section 15. Show that the real limits can not exist – does this approach give another rigorous proof of the non-existence of the complex-valued limit?