Homework #4 • MATH 322 • A First Look at Analyticity

- submit your write-up into your Section's box by noon, Friday 06 October.
- please acknowledge collaborations & assistance from colleagues.
- have a happy Thanskgiving.

A) **A Square Root Paradox** (2 pages max, 10pts) This exercise is based upon problem #2 on page 84. Based upon the definition of analytic continuation found on page 82, explain why $f_2(z)$ is the analytic continuation of $f_1(z)$. What is the domain of analyticity of the function obtained by combining these results? (You may ignore the rest of problem #2.) What is the value at $z = -i$?

Now consider a slightly modified version of $f_1(z)$ on a domain D that now includes the positive real line

$$D = \{z \mid 0 < r < \infty, \ 0 \le \theta < \pi\}$$

(call this function $f_D(z)$). Apply the reflection principle to extend the domain of analyticity. What is the domain of analyticity of the function obtained through this extension? What is the value at $z = -i$?

Describe in one sentence why these results might seem paradoxical?

Bonus: At this point, I do not expect you to be able to resolve the paradox illustrated by the above calculation. However for a bonus, make a brief comment on whether or not you think these results contradict the (future) theorem of page 81. Give reason(s) for your position.

B) **Exponential & Logarithm** (1 page for each problem, 10pts) Problems #4 on page 89, #12 on page 90, and #5 on page 94. Give two reasons for the question of harmonicity in #12 – you may quote your results from your modification of the maple script *hw04B.mw* from the class webpage.

C) **Logarithms** (2 pages max, 10pts) In addition to finding the principal value for problem #2c on page 99, list all possible values. Also, present problems #8b, 8c and 9 on page 100.