Homework #9 • MATH 322 • Analyticity of Series & Residues

- submit your write-up into your Section's box by noon, Friday 24 November.
- please include your SFU login name with your name on the assignment.

A) Removable Singularities (10 pts, 2 pages) Review example 1 of section 59 (Integ. & Diff. of P. Ser.). Then construct a parallel argument for the analyticity of the function as defined by Problem #5 on page 213. Be clear about the regions of analyticity of all functions (and representations) involved in the argument.

B) Reciprocal Series (10 pts, 3 pages) The text suggests a *synthetic* division approach to series quotients in section 61. I prefer direct application of equation (4) on page 216 for calculating the reciprocal of a series $g(z) = 1/f(z)$ which begins from

$$\sum_{n=0}^{\infty} b_n\ z^n = g(z) = \frac{1}{f(z)} = \left\{ \sum_{n=0}^{\infty} a_n\ z^n \right\}^{-1}$$

and then demands that

$$\left\{ \sum_{n=0}^{\infty} b_n\ z^n \right\} \left\{ \sum_{n=0}^{\infty} a_n\ z^n \right\} = 1\ .$$

The right-side of the above is a one-term series, and by uniqueness of series, we can equate coefficients after using equation (4) for the left-side. Since the a_n-coefficients are given, it turns out that the b_n-coefficients can be solved for sequentially (provided $a_0 \neq 0$). Apply this idea to Problem #3 on page 219.

(Corrected version.)

The above Laurent series gives a value for the integral

$$\oint_{\mathcal{C}} \frac{1}{e^z - 1}\ dz$$

where $\mathcal{C}$ is the unit circle around the origin (use the parametrization, $z = e^{i\theta}$). Deriving explicit expressions for the real and imag parts of the integrand gives two (rather intimidating) real-valued, definite integrals — which you now know how to evaluate! Could you have obtained either of these results using your pre-requisite knowledge from real-valued calculus?

C) Residues (15 pts, 3 pages) Problems #2b, 2d and 3a on page 230. For the problems #2b and 2d, present two methods for obtaining the desired residues.

***) Other Problems** (optional) Problems # 7 and 10 on page 214. Problems # 1 and 5 on page 230.