Homework #0 • MATH 322 • Warm-up: Arithmetic of Complex Numbers

- read the *Guideline* for assignments as posted on the class webpage.
- please respect the page limit.
- submit your write-up into the math322 box by noon, Wednesday 12 September.
- remember that webct is an open forum for discussion.
- collaborations & assistance from colleagues are permitted on assignments, but you must acknowledge help from others, including TAs, etc.

A) Student Info Form (on webct & class webpage) Please fill out & attach as the front page of your submitted assignment. If you have already submitted this form in class, you are welcome to fill out another version, or leave the grader a note asking to find yours in the stack.

B) Square Roots of a Complex Number (2 pages max, 10pts) Given a complex number $z = x + iy$, define a square root of z as any complex number w such that $w^2 = z$. Find formulas for all of the square roots of $x + iy$. Begin by expressing w as $a + ib$ and carefully (explain why your arithmetic is valid) solve for the real values of $a(x, y)$ and $b(x, y)$ – how many distinct square roots did you expect & how many are implied by your final formula? Check your answer by finding all of the square roots of $1 - 7i$.

***) Extra Problems to Note** (optional) For those who are concerned about their understanding of the basics of complex numbers, I consider the problems listed below as introductory material for this class. Please read through them, you may offer them up as a discussion the in first tutorial if you wish.

- #5, 7; page 5
- #1; page 8
- #2, 4, 5; page 12
- #1, 2, 10; page 14-15
- #1, 2, 5; page 22-23

***) Bombelli's (1526-73) Vision** (optional challenge problem) Cardano's (1501-76) application of Tartaglia's (1500-77) formula for the solution of the cubic polynomial $x^3 = 15x + 4$ gave the odd-looking result

$$x = \sqrt[3]{2 + \sqrt{-121}} + \sqrt[3]{2 - \sqrt{-121}}$$

when in fact, it was known that there was a perfectly fine integer root. Invoking the rules of complex arithmetic, show that the above expression can indeed be a simple, real-valued root. (Hint: Bombelli had the insight to guess the complex-valued cube roots.)