Homework -1 • MATH 322 • Prerequisite Mathematics

- not for submission, but may be addressed in tutorials if time permits.
- you are encouraged to reference a calculus textbook to remind you about details.
- these questions represent an overview of the language of the calculus that features prominently in the complex variables course, math 322.

REMINDER QUESTIONS.

7–19 Prove the identity.

7. $\sinh(-x) = -\sinh x$
(This shows that sinh is an odd function.)

8. $\cosh(-x) = \cosh x$
(This shows that cosh is an even function.)

HYPERBOLIC

9. $\cosh x + \sinh x = e^x$

11

10. $\cosh x - \sinh x = e^{-x}$

11. $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$

12. $\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$

30–45 Find the derivative. Simplify where possible.

33

30. $f(x) = \tanh(1 + e^{2x})$ **31.** $f(x) = x \sinh x - \cosh x$

32. $g(x) = \cosh(\ln x)$ **33.** $h(x) = \ln(\cosh x)$

34. $y = x \coth(1 + x^2)$ **35.** $y = e^{\cosh 3x}$

LIMIT

40 (squeeze)

39. Prove that $\lim_{x \to 0} x^4 \cos \frac{2}{x} = 0$.

40. Prove that $\lim_{x \to 0^+} \sqrt{x}\, e^{\sin(\pi/x)} = 0$.

CONTINUITY

30

29–38 Determine the set of points at which the function is continuous.

29. $F(x, y) = \dfrac{xy}{1 + e^{x-y}}$ **30.** $F(x, y) = \cos\sqrt{1 + x - y}$

31. $F(x, y) = \dfrac{1 + x^2 + y^2}{1 - x^2 - y^2}$ **32.** $H(x, y) = \dfrac{e^x + e^y}{e^{xy} - 1}$

55–59 Find the derivative of the function.

55. $g(x) = \displaystyle\int_{2x}^{3x} \frac{u^2 - 1}{u^2 + 1}\, du$

$$\left[\textit{Hint: } \int_{2x}^{3x} f(u)\, du = \int_{2x}^{0} f(u)\, du + \int_{0}^{3x} f(u)\, du \right]$$

FUND. THM OF CALC

55

56. $g(x) = \displaystyle\int_{1-2x}^{1+2x} t \sin t\, dt$

7–10 Find the exact length of the curve.

7. $y = 1 + 6x^{3/2}, \quad 0 \leqslant x \leqslant 1$

8. $y^2 = 4(x + 4)^3, \quad 0 \leqslant x \leqslant 2, \quad y > 0$

9. $y = \dfrac{x^3}{3} + \dfrac{1}{4x}, \quad 1 \leqslant x \leqslant 2$

ARCLENGTH

#7. #45

45–48 Find the exact length of the polar curve.

45. $r = 2\cos\theta, \quad 0 \leqslant \theta \leqslant \pi$

46. $r = 5^\theta, \quad 0 \leqslant \theta \leqslant 2\pi$

47. $r = \theta^2, \quad 0 \leqslant \theta \leqslant 2\pi$

48. $r = 2(1 + \cos\theta)$

SERIES.

#31 (geometric)

#11 (also geometric!)

27–42 Determine whether the series is convergent or divergent. If it is convergent, find its sum.

27. $\dfrac{1}{3} + \dfrac{1}{6} + \dfrac{1}{9} + \dfrac{1}{12} + \dfrac{1}{15} + \cdots$

28. $\dfrac{1}{3} + \dfrac{2}{9} + \dfrac{1}{27} + \dfrac{2}{81} + \dfrac{1}{243} + \dfrac{2}{729} + \cdots$

29. $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{3n-1}$

30. $\displaystyle\sum_{k=1}^{\infty} \frac{k(k+2)}{(k+3)^2}$

31. $\displaystyle\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$

32. $\displaystyle\sum_{n=1}^{\infty} \frac{1+3^n}{2^n}$

11–12 Express the function as the sum of a power series by first using partial fractions. Find the interval of convergence.

11. $f(x) = \dfrac{3}{x^2 - x - 2}$

12. $f(x) = \dfrac{x+2}{2x^2 - x - 1}$

TAYLOR SERIES (at $x = 0$)

#5

5–12 Find the Maclaurin series for $f(x)$ using the definition of a Maclaurin series. [Assume that f has a power series expansion. Do not show that $R_n(x) \to 0$.] Also find the associated radius of convergence.

5. $f(x) = (1 - x)^{-2}$

6. $f(x) = \ln(1 + x)$

7. $f(x) = \sin \pi x$

8. $f(x) = e^{-2x}$

9. $f(x) = 2^x$

10. $f(x) = x\cos x$

11. $f(x) = \sinh x$

12. $f(x) = \cosh x$

3

SURFACES & LEVEL CURVES

59–64 Match the function (a) with its graph (labeled A–F below) and (b) with its contour map (labeled I–VI). Give reasons for your choices.

59. $z = \sin(xy)$

60. $z = e^x \cos y$

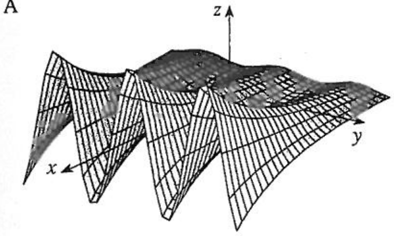
61. $z = \sin(x - y)$

62. $z = \sin x - \sin y$

63. $z = (1 - x^2)(1 - y^2)$

64. $z = \dfrac{x - y}{1 + x^2 + y^2}$

A



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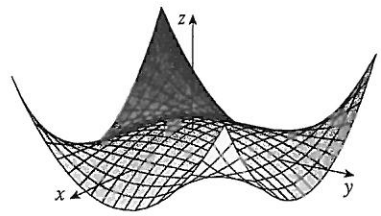

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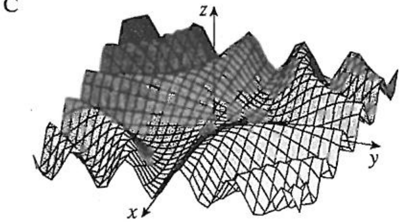

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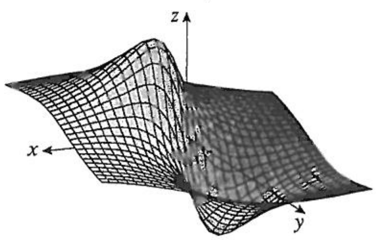

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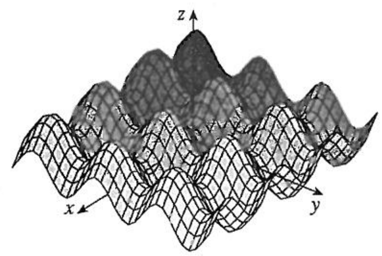

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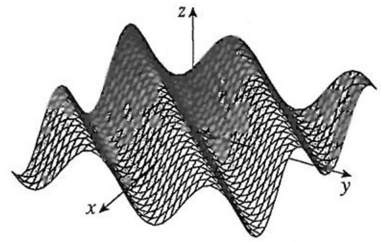
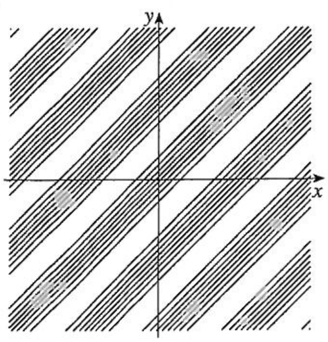


I



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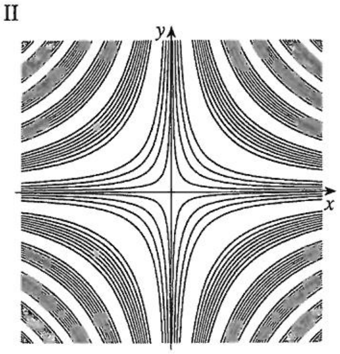
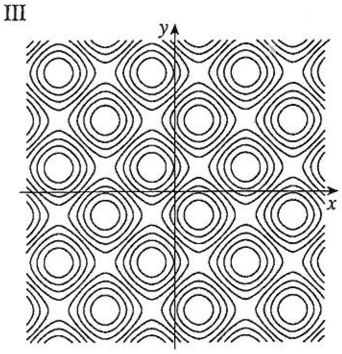


III



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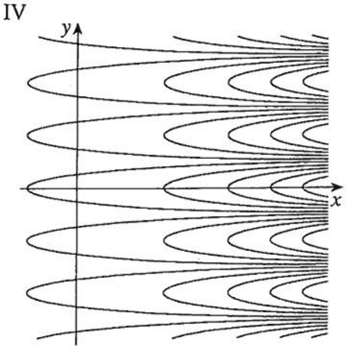

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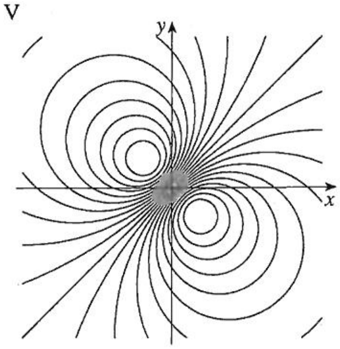
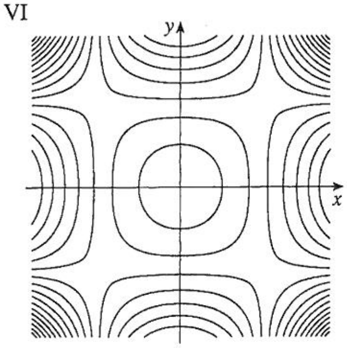


VI



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19. Given that f is a differentiable function with $f(2, 5) = 6$, $f_x(2, 5) = 1$, and $f_y(2, 5) = -1$, use a linear approximation to estimate $f(2.2, 4.9)$.

20. Find the linear approximation of the function $f(x, y) = 1 - xy \cos \pi y$ at $(1, 1)$ and use it to approximate $f(1.02, 0.97)$. Illustrate by graphing f and the tangent plane.

21. Find the linear approximation of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at $(3, 2, 6)$ and use it to approximate the number $\sqrt{(3.02)^2 + (1.97)^2 + (5.99)^2}$.

LINEAR APPROX

#21

DIRECTIONAL DERIV

#11

11–17 Find the directional derivative of the function at the given point in the direction of the vector $\mathbf{v}$.

11. $f(x, y) = e^x \sin y, \quad (0, \pi/3), \quad \mathbf{v} = \langle -6, 8 \rangle$

12. $f(x, y) = \dfrac{x}{x^2 + y^2}, \quad (1, 2), \quad \mathbf{v} = \langle 3, 5 \rangle$

6 Find the Jacobian of the transformation.

5) $x = 5u - v, \quad y = u + 3v$

$x = uv, \quad y = u/v$

$x = e^{-r} \sin \theta, \quad y = e^r \cos \theta$

VARIABLE CHANGES

#5

DOUBLE INT.

#17

17–22 Evaluate the double integral.

17. $\iint_D x \cos y \, dA, \quad D$ is bounded by $y = 0, \ y = x^2, \ x = 1$

18. $\iint_D (x^2 + 2y) \, dA, \quad D$ is bounded by $y = x, \ y = x^3, \ x \geqslant 0$

LINE INTEGRAL

#3

1–16 Evaluate the line integral, where C is the given curve.

1. $\int_C y^3 \, ds, \quad C: x = t^3, \ y = t, \ 0 \leqslant t \leqslant 2$

2. $\int_C xy \, ds, \quad C: x = t^2, \ y = 2t, \ 0 \leqslant t \leqslant 1$

3. $\int_C xy^4 \, ds, \quad C$ is the right half of the circle $x^2 + y^2 = 16$

4. $\int_C x \sin y \, ds, \quad C$ is the line segment from $(0, 3)$ to $(4, 6)$