Homework #1 • Numerical Analysis II (math 416) • Finite Difference Solution for a BVP

- due Wednesday 20 September.
- remember that the class e-mail is open for discussion.
- *annotate* all plots (can be handwritten on plots). — page limits include annotated plots.

A) (2 pages) Download the script *lect03.m* for solving the 2-point BVP

$$y'' - \sigma^2 y = x \qquad \text{on } \; 0 \le x \le 1 \qquad \text{with } \; y(0) = y(1) = 0$$

for $\sigma = 1$. Verify that, in fact, the matlab LU decomposition as invoked by the "\" operator leads to an operation count that scales as $O(N^2)$. (Hint: plotting what quantities will result in a meaningful graphic?)

B) (2 pages) Replace, in the script, the one-line LU solve with two loops that implement the exact LU decomposition as presented in class. Solving the two sparse triangular systems

$$\mathbf{L}\,(z_j) = (x_j) \qquad ; \qquad \mathbf{U}(y_j) = (z_j)$$

requires two separate loops. The basic structure has the logical *pseudo-code*:

set d_1
solve z_1
loop 1: $j = 2 \to N-1$
  set m_j
  set d_j
  solve z_j
loop 2: $j = N-1 \to 1$
  solve y_j

where the notation follows from the lecture. Verify that the operation count of your modified code indeed scales as $O(N)$. Remember to verify the second-order convergence of the error.

C) (2 pages) The purpose of this exercise is to use the numerical solver to investigate the properties of the BVP. Modify your script to replace $\sigma^2 \to K$ – so that you can make K zero, or even negative, if you wish. Choose one of the following:

(i) What is the exact solution of the limiting BVP as $K \to 0^+$ (limit from above)? What is the limit as $K \to 0^+$ of the exact solution to the BVP? (These questions are different in the order that the BVP is solved & the limit is taken.) Finally, is the numerical code well-behaved in this limit? Reconcile all three approaches to this problem.

(ii) Investigate the solution for the interval $-10 \le K \le -9.8$ — note the existence of a critical value. What do you think makes this value of K so special? (Hint: check the conditioning of the matrix.)