Homework #7 • Numerical Analysis II (math 416) • Timestepping for PDEs

- due Wednesday 15 November.
- quality of presentation will be counted heavily this assignment.

A) (2 pages) Find the general term of the vector iteration

$$\vec{U}^{k+1} = \begin{bmatrix} 0 & 1-\lambda & 0 & 1+\lambda \\ 1+\lambda & 0 & 1-\lambda & 0 \\ 0 & 1+\lambda & 0 & 1-\lambda \\ 1-\lambda & 0 & 1+\lambda & 0 \end{bmatrix} \vec{U}^k \equiv \mathbf{M}(\lambda)\, \vec{U}^k$$

when $\lambda = 1/2$ and $\vec{U}^0 = (1\ 2\ 3\ 4)^T$. Derive the formula using exact eigenvectors (not Matlab calculated). What is the set of initial vectors $\vec{U}^0$ for which the iteration decays as $k \to \infty$?

B) (4 pages) Read the attached page describing the Lax-Wendroff scheme. Implement this scheme to numerically solve the forced one-way wave equation

$$u_t + u_x = f(x,t) \quad \text{on} \quad 0 \le x \le 2\pi\ ,\ 0 \le t$$

with zero initial conditions, u(x,0) = 0, and where the forcing function is given by

$$f(x,t) = e^{-(x-\pi)^2-(t-\pi)^2} - e^{-(x-\pi)^2-(t-3\pi)^2}\ .$$

What seems to be happening to the numerical solution as t get large? Can you explain why this should be so? (Hint: use the linearity of the PDE.)

Show evidence that your code displays the proper convergence.