Homework #0 • MATH 418 • Warm-Up Problems

- please respect page limits & practice the *Guidelines for Written Assignments.*
- submit your write-up by midnight, Wednesday 16 September (homework box #12).
- remember that Webct is an open forum for discussion.
- acknowledge collaborations & assistance from colleagues/instructor.

- student info form attached. Submit with assignment if you haven't already done so.

- consider the warm-up problems below.
- identify which problems you **cannot** do.
- of these problems that you **can** do, submit 1-page presentations for the three that you found most challenging.
- state references – not everything in your summaries need be derived. You may certainly discuss references on Webct.

A) Polar Coordinates (1 page) Verify by direct calculation that the function $u(r, \theta) = \cos(r \cos \theta)$ satisfies the PDE relation

$$\nabla^2 u + u = 0 \ .$$

Investigate from the three different perspectives:

(a) converting to rectangular coordinates: $u(r, \theta) = U(x, y)$;

(b) using the polar form of the Laplacian operator; and

(c) evaluating u_{xx} and u_{yy} from $u(r, \theta)$ by chain rule.

B) Complex Variables (1 page) Find the complex-analytic function, $f(z)$, on the unit disc that takes the values

$$f(z) = -3i\, e^{3i \arg z} + (1 - 2i)\, e^{-i \arg z}$$

on the unit circle $|z| = 1$. Give an explicit formula for the real part of the function, $u(x, y) = \mathrm{Re}\{f(z)\}$ where $z = x + iy$. Calculate the Laplacian of $u(x, y)$, and state the theorem from which this result derives.

C) Continuity (1 page) Consider the parabola function $y = x^2$ on $0 \le x \le 1$. Using the standard ϵ–δ definition of a continuous function, give the *best* formula for $\delta(\epsilon; x)$ that guarantees continuity at the point x. Give the best value of $\tilde{\delta}(\epsilon)$ that guarantees uniform continuity on the interval $0 \le x \le 1$.

D) Vector Calculus (1 page) Explain why the area of a 2D region bounded by a simple closed curve, $\mathcal{C}$, has an area formula in the form of the line integral

$$A = \frac{1}{2} \int_{\mathcal{C}} \begin{pmatrix} x \\ y \end{pmatrix} \cdot \hat{n}\ ds$$

(there are, in fact, two such explanations). Derive the area for a semi-circle using the above formula.

E) ODE Boundary Value Problem (1 page) Find a closed form solution, $q(x)$, for the ODE problem on $0 \leq x \leq 1$

$$\frac{d^2 q}{dx^2} = f(x) \qquad \text{with} \qquad q(0) = q(1) = 0 \ .$$

Show that the use of integration by parts (followed by some careful bookkeeping), the double integral solution can be reduced to a single integral formula having the form

$$q(x) = \int_0^1 f(s)\, G(x,s)\ ds \ .$$

(Hint: the function $G(x,s)$ is defined piecewise on $0 \leq s \leq 1$.)

F) Linear Algebra (1 page) Consider the matrix equation for the vector $\vec{v}$

$$\begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix} \vec{v} \ = \ \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \ .$$

Construct vectors $\vec{w}_j$ such that $\vec{v}$ has a representation in the form

$$\vec{v} = \sum_{j=1}^{3} a_j\, \vec{w}_j \ .$$

What is the significance of the vectors $\vec{w}_j$?

NAME & Places: (hometowns, etc)

Year & Programs: (4^{th} year MATH/APMA, for example)

E-Mail (req) & Local Phone (opt):

Quantitative Courses: (term taken & text)

linear algebra & diff. equations

adv. calculus & analysis

courses with computing

other quant courses (sciences, engineering, economics, etc)

Matlab & Maple – Experience: (yes/no)

Matlab & Maple – Access: (lab and/or home)

Other Computing Experience: (software, programming languages, web design)

Subjects of Interest: (specific areas of math, sciences, etc)

Mathematical Focus: rank in order of priority (1 = most, 3 = least)

[ ] analysis/theory [ ] applications [ ] computing & graphics

Personal Course Objectives: goals for this class & future plans

Familiarity Scale: I know it ...

- **5** ... in my sleep!
- **4** ... after a bit of thinking
- **3** ... should I see it in class again
- **2** ... if I can wikipedia it
- **1** ... vaguely from a previous exam question I couldn't answer
- **0** ... huh?
- **-7** ... is a subject to be avoided at all costs

Mathematical Topics: use above scale

- ☐ CALC: implicit (partial) differentiation
- ☐ CALC: multi-variable chain rule & change of variables
- ☐ CALC: multiple integrals
- ☐ CALC: theorems of Green & Stokes
- ☐ LIN ALG: solution methods for systems of linear equations
- ☐ LIN ALG: existence & uniqueness of solutions for systems of linear equations
- ☐ LIN ALG: matrix eigenvalues & eigenvectors
- ☐ ODEs: solution methods for 2^{nd}-order linear ODEs
- ☐ ODEs: using initial conditions for 2^{nd}-order linear ODEs
- ☐ ODEs: solution of linear ODE systems
- ☐ ODEs: eigenvalues & eigenfunctions
- ☐ SERIES: deriving Fourier series
- ☐ SERIES: solution of BVPs by Fourier series
- ☐ COMPLEX: complex exponential notation
- ☐ COMPLEX: complex contour integration
- ☐ COMPLEX: Fourier transform integrals