PDEs: Solutions, Properties & Applications

Many areas of mathematics and the sciences involve functions of more than one variable that are defined by the solution of an equation relating its partial derivatives. This introduction to the theory of partial differential equations (PDEs) begins with the trilogy of the basic linear prototypes, known as:

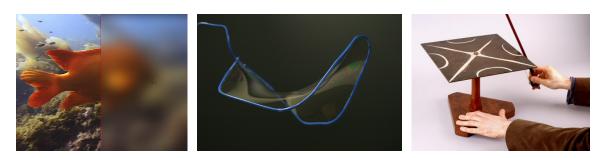
- the Laplace & Poisson equations,
- the diffusion equation, and
- the wave & Helmholtz equations.

Linear theories are developed by combining familiar ideas from multi-variable calculus, linear algebra, and ordinary differential equations. We then obtain a variety of analytical methods for the solution and understanding of these equations. More abstract notions of existence, uniqueness, and solution properties are also explored. Finally, our study of these PDEs includes brief discussions of their origins and simple applications. As examples, probabilities can be understood in terms of diffusion, sound propagation can be described by waves, and gravitational forces are derivable from Poisson potentials.

Time and interest permitting, additional topics could include: Fourier series and transform methods, nonlinearity and shock waves, calculus of variations and optimization.

Calendar course prerequisites: Math 251 (multi-variable calculus) & Math 310 (ordinary differential equations) contain most of the foundational ideas. Elementary PDE ideas from Math 314/Phys384, and analysis experience from Math 320 and 322 are advantageous, but not essential. Some familiarity with Maple and/or Matlab computing is helpful.

Further information & updates: www.math.sfu.ca/~muraki



These images are depictions of the three basic linear PDEs. The Gaussian blurring of images is a 2D application of diffusion. The slight geometrical distortions of a soap film can be described by the Laplace equation. An acoustic mode of a flat plate is visualized by a Chladni pattern that is related to solutions of the Helmholtz PDE.