Homework #4 • MATH 419 • Convergence of Fourier Series

- submit your write-up Wednesday 08 June.

A) Pointwise Convergence (3 pages) Present a rigorous proof for the limit of the convolution with the Dirichlet kernels

$$\lim_{N\to\infty} \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x+\theta)\, D_N(\theta)\; d\theta \tag{1}$$

where $-\pi < x < +\pi$ is a point of discontinuity, but both $f(x)$ and $f'(x)$ have continuous left and right limits. The function may be assumed to be piecewise continuous everywhere else on $[-\pi, +\pi]$. You may quote an ϵ-δ result from the Riemann-Lebesgue lemma (since you've already proved this) whenever it is needed. Note: a short discussion of this appeared in Monday's lecture.

B) Bessel's Inequality (2 page) Derive the cos / sin version of the Bessel inequality. In your opinion (give supporting explanation), how many key steps are required in the proof?

C) Césaro Convergence (3 pages) Devise an efficent formulation to compute the Césaro average. Modify the matlab script *w04one.m* to compare the convergence properties of Fourier partial sums and their Césaro averages for the function

$$f(x) = \begin{cases} -1 & \text{for } -\pi < x < 0 \\ +1 & \text{for } 0 < x < +\pi \\ 0 & \text{for } x = 0, \pm\pi \end{cases} \tag{2}$$

Make some observations and give quantitative support for them (webct discussions appropriate here).