Homework #5 • MATH 419 • Discrete Fourier Series

- submit your write-up Wednesday 29 June.

A) Discrete Convolution (2 pages) Present the proof for the convolution part of Theorem 3.4 in the text (Exercise 3.3.2). To be consistent with the Matlab convention, define the convolution with the indexing

$$(y * z)_J = \sum_{j=1}^{N} y_j \, z_{J-j+1} \; . \tag{1}$$

The algebraic manipulations are straightforward, but explain very carefully the limits of your sums (I know it works out, but you have to be clear about why it does). Also prove the symmetry property of the convolution $y * z = z * y$. (Note that the sum of the subscripts is a good way to remember the convention for the convolution index.)

B) Diagonalizing the Circulant Matrix (3 pages) Parts (b) and (c) of Exercise 3.3.15 in the text. With our indexing convention, I believe the easiest notation for part (a) begins with the following definition of **A**

$$\mathbf{A} \; = \begin{bmatrix} a_1 & a_N & \dots & a_2 \\ a_2 & a_1 & \dots & a_3 \\ \vdots & \vdots & \ddots & \\ a_N & a_{N-1} & \dots & a_1 \end{bmatrix} \; . \tag{2}$$

The diagonalization of part (c) in the notation of Wednesday's lecture would be

$$\frac{1}{N} \, \mathbf{W}_N^H \, \mathbf{A} \, \mathbf{W} = \mathbf{\Lambda} \; . \tag{3}$$