MATH 462 - Homework #0 Solution - by Ben Ong

Finding Stagnation Points (Vector Calculus)

Given a scalar field

$$\phi(x,y) = y\left(1-\frac{1}{r^2}\right)+\frac{B}{2}\ln\left(r^2\right) \text{ where } r^2 = x^2+y^2$$

Define a vector field $\overrightarrow{U}(x,y) = ((u(x,y), v(x,y))$ where

$$u(x,y) = \frac{\partial \phi}{\partial y} = \left(1-\frac{1}{r^2}\right)+\frac{2y^2}{r^4}+\frac{By}{r^2} = \left(1-\frac{1}{r^2}\right)+\frac{2y^2}{r^2}\left(\frac{y^2}{r^2}+\frac{B}{2}\right) \tag{1}$$

$$v(x,y) = -\frac{\partial \phi}{\partial x} = \frac{2xy}{r^4}+\frac{Bx}{r^2} = \frac{2x^2}{r^2}\left(\frac{y}{r^2}+\frac{B}{2}\right) \tag{2}$$

We wish to find when $\overrightarrow{U}(x,y) = (0,0)$. Notice that $v = 0$ when

<u>Case 1</u>, $x = 0$

Setting equation (1) to zero (with simplification) results in

$$u(x,y) = 1-\frac{1}{y^2}+\frac{2}{y^2}+\frac{B}{y} = 0$$

or equivalently

$$x = 0, \qquad y = \frac{-B}{2} \pm \frac{\sqrt{B^2-4}}{2} \qquad \text{which are real provided } |B| \geq 2 \tag{3}$$

<u>Case 2</u>, $\frac{y}{r^2}+\frac{B}{2} = 0 \Rightarrow y = -\frac{Br^2}{2}$

Setting equation (1) to zero (with simplification) results in the condition $r^2 = 1$. Thus

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$$y = -\frac{B}{2}, \qquad x = \pm\sqrt{1-\frac{B^2}{4}} \qquad \text{which are real provided } |B| \leq 2 \tag{4}$$
