Homework #4 • MATH 462 • Potential Flow

- submit your write-up Wednesday 13 February.
- please acknowledge collaborations & assistance from colleagues.
- midterm reminder: Wednesday 27 February.

A) Pie Flow (3 pages, 10pts) Find all separable solutions to the Laplace equation which produce a streamfunction for the (polar) sector $0 \leq r < \infty$ and $0 \leq \theta \leq 2(\pi - \alpha)$. As one BC, the walls of the sector must constitute a streamline, which for simplicity, can be taken to have zero streamfunction (why?). If no BC is imposed at $r \to \infty$, then the solutions will be arbitrary up to a multiplicative constant.

Invert the Cauchy-Riemann equations to give the velocity potential, and construct the complex potential $\Phi(z)$. Is the complex potential an analytic function? Explain. Give simple expressions for the flow velocities.

B) Pie Flow with a Bite? (2 pages + plots, 10pts) Plotting the solution from part **A)** using Matlab can be a little tricky. Consider the simplest two "$n = 1, 2$" solutions. If you simply evaluate the complex potential in *code05a.m*, you do NOT get the picture you want (how very annoying). The problem is that $\Phi(z)$ not uniquely defined, but is multiple-valued! Matlab doesn't make the plot you want because it happens to choose the wrong value (for those with complex experience, this essentially the branch cut thing). To get what you want, you need to have Matlab compute exactly what you want. Explain how the code snippets:

$$[t1, r1] = cart2pol[xx, yy]; \quad t2 = mod(t1, 2 * pi);$$

produce polar coordinate matrices $\theta \to t2$ and $r \to r1$ which have the useful values. If you now evaluate the streamfunction and velocity potential explicitly, the resulting plot works out well.

Overlay a contourplot of pressure for several choices of angle α, and note when and what physical fields can be singular at the origin. Choose your plots wisely.

For a value of $0 \leq \alpha \leq \pi/4$, show how your "$n = 2$" solution is modified if the tip is dulled by changing the boundary to a finite radius arc R so that $0 \leq R \leq r < \infty$.

C) (3 pages, 10pts) Read Chapter 4 and present a discussion based upon problem 4.3 in Acheson.