Homework #7 • MATH 462 • Sound & Viscosity

- submit your write-up Wednesday 20 March.
- thank you for acknowledging collaborations & assistance from colleagues.

A) Music of the Sphere? (2 pages, 10pts) This problem is based on #3.13 in Acheson. Derive the linear wave equation for a spherically symmetric wave from the Euler equations. For a spherical shell of radius L, derive the eigenvalue relation for the natural (temporal) frequencies, ω, of the interior standing waves

$$\tan \frac{\omega L}{c} = \frac{\omega L}{c} \ .$$

Explain why the boundary conditions of bounded density at the origin and zero velocity at $r = L$ are reasonable choices. Calculate (approximately) the first three eigenfrequencies (as multiples of c/L), and give an opinion on whether or not this *music of the sphere* is a truly harmonious sound.

B) Cylindrical Navier-Stokes (2 pages, 10pts) Show how the viscous terms expressed in cylindrical coordinates are derived using the two approaches discussed in section 2.4 of Acheson. Which approach gives a clearer presentation?

C) Turning on the Water (3 pages, 10pts) Solve the problem as posed by #2.5 in Acheson. There are several ways to approach the mathematics, some maybe more efficient & pleasant than others. Use Matlab to calculate at what time the flow speed at $r = 0$ reaches half of the (steady) Couette flow value (state your choices for parameter values).