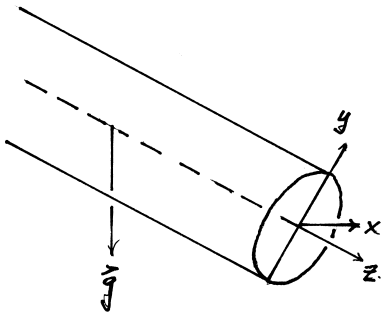
Homework #8 • MATH 462 • Last Questions

- write-ups are due over two Wednesdays, 27 March and 03 April.
- assignment for 27 March should be submitted to an envelope at the math office front desk.
- presentation is a very important aspect of these final problems.

A) Pipe Poiseuille Flow (due 27 March, 4 pages, 15pts) Use the steady Navier-Stokes equations to calculate *gravity-driven* flow through a pipe which is inclined at an angle α from the horizontal. I recommend using cylindrical coordinates with the $\hat{z}$-axis oriented with the pipe axis – hence, gravity will be seem to be *tilted* in the equations.



Show that in the above coordinate geometry

$$\vec{g} = -\cos\alpha\ (\sin\theta\,\hat{r} + \cos\theta\,\hat{\theta}) + \sin\alpha\,\hat{z}\ .$$

It is not surprising that the axial velocity is z-independent, but it turns out also to be θ-independent (like usual pipe Poiseuille flow). Thus $W(r)$ only. The pressure is also z-independent, but develops a cross-sectional distribution.

Calculate the mass flux through the pipe. Plot the cross-sectional pressure distribution.

B) Annular Shear Flow (due 27 March, 3 pages, 15pts) Consider a viscous 2D fluid in an annulus $A < r < B$ where a shear flow is driven by rotating the center boundary cylinder with an angular velocity of Ω (radians/time). Calculate the interior flow field.

C) Spinning Sphere (due 03 April, 5 pages, 20pts) Solve the problem as posed by #7.2 in Acheson. This is the final write-up and presentation (with discussion) will be graded seriously. Show that you understand both the mathematics and the fluid dynamics. (Presenting just equations will earn only a 1/4 grade. Key words are: correctness, clarity & conciseness.)