Homework #1 • MATH 462 • The Euler Equations

- please respect page limits.
- submit your write-up by noon, Thursday 22 January.
- remember that webct is an open forum for discussion.
- please acknowledge collaborations & assistance from colleagues.

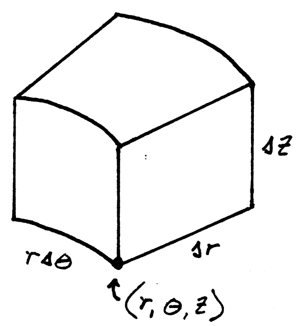
A) Polar Coordinates (4 pages, 20pts) Present a *first principles* derivation of the Euler equations for two-dimensional fluid flow in three-dimensional cylindrical coordinates (r, θ, z). In this context, the two-dimensional flow is the special case with no vertical flow ($w \equiv 0$) and no vertical variations ($\partial/\partial z \equiv 0$). For convenience, define the velocities $U(r, \theta, t)$ and $V(r, \theta, t)$ to be the flow components in the $\hat{r}$ and $\hat{\theta}$-directions. Invoking

(a) conservation of mass, and

(b) Newton's law with a given body force (per unit volume) $\vec{F}(r, \theta, t)$;

should result in three PDEs for the flow velocities, density $\rho(r, \theta, t)$ and pressure $p(r, \theta, t)$.

Quality (clarity & conciseness) of the presentation counts. Please use a few words to explain your reasoning, and use several line diagrams to accompany your derivation.



Added calculus note: Remember that there is a θ-dependence of the unit basis vectors $\hat{r}(\theta)$ and $\hat{\theta}(\theta)$.

$$\hat{r}(\theta + \triangle\theta) \approx \hat{r}(\theta) + \hat{\theta}(\theta)\triangle\theta \quad ; \quad \hat{\theta}(\theta + \triangle\theta) \approx \hat{\theta}(\theta) - \hat{r}(\theta)\triangle\theta \; .$$

bonus: Use Appendix A.6 to verify that your conservation of mass equation is consistent with the expected result in Cartesian coordinates.

B) The Spinning Bucket Problem (2 pages, 10pts) Consider the flow velocity for a uniformly rotating fluid $(u, v, w) = (-\Omega y, \Omega x, 0)$. Find the accompanying pressure field $p(x, y, z)$ which produces a flow solution to the *incompressible* Euler equations in the presence of gravity $\vec{F} = -\rho g \hat{z}$. Having done this, now discuss and resolve the *confusion* suggested in the first two paragraphs of Problem 1.2 (Acheson) involving the Bernoulli streamline theorem (Section 1.3). Why might an astronomer with a stash of mercury find this to be an interesting result?