Homework #4 • MATH 462 • Potential Flow

- submit your write-up 1pm, Friday 13 February.
- please acknowledge collaborations & assistance from colleagues.

A) Pie Flow (4 pages max, 10pts) Apply the method of separation of variables to the Laplace equation to produce all possible streamfunctions, $\psi(x, y)$, in the (polar) sector $0 \leq r < \infty$ and $0 \leq \theta \leq 2(\pi - \alpha)$. Take as one BC, $\psi = 0$, since the walls of the sector must constitute a streamline (why?). If no BC is imposed at $r \to \infty$, then the solutions will be arbitrary up to a multiplicative constant.

Invert the Cauchy-Riemann equations to give the velocity potentials, and construct the complex potentials $\Phi(z)$. Is the velocity potential analytic? Explain. Give simple expressions for the flow velocities and (dynamic) pressure. Note as a function of α which fields and solutions are singular at the origin.

Extra plots: Plot some of the flows. Also, for a value of $0 \leq \alpha \leq \pi/4$, show how your "$n = 2$" solution is modified if the tip is dulled by changing the boundary to a finite radius arc R so that $0 \leq R \leq r < \infty$.

B) Flow Plots (1 page + plots, 10pts) Plotting flow solutions with singularities can be tricky in Matlab. Consider two ways of calculating the same streamfunction:

$$\Phi_1(z) = \frac{z}{c} + \frac{1}{2\pi} \log\left(\frac{z+c}{z-c}\right)$$

$$\Phi_2(z) = \frac{z}{c} + \frac{1}{2\pi} \log(z+c) - \frac{1}{2\pi} \log(z-c)$$

where $c = e^{i\alpha}$ where α is an angle. Describe the flow, and compare the plotted results when c takes a complex value. Explain why the plots are different.

C) Use the Force (3 pages, 10pts) Read Chapter 4 and present a discussion based upon problem 4.3 in Acheson.