Homework #8 • MATH 462 • Sound & Vorticity

- submit your write-up Friday 19 March.

A) Music of the Sphere? (2 pages, 10pts) This problem is based on #3.13 in Acheson. Derive, from the compressible Euler equations the linear wave equation for a spherically symmetric sound wave . For a spherical shell of radius L, derive the eigenvalue relation for the natural (temporal) frequencies, ω, of the interior standing waves

$$\tan \frac{\omega L}{c} = \frac{\omega L}{c} \ .$$

Explain why the boundary conditions of bounded density at the origin and zero velocity at $r = L$ are reasonable choices. Calculate (approximately) the first three eigenfrequencies (as multiples of c/L), and give an opinion on whether or not this *music of the sphere* is a truly harmonious sound.

(Also, do you understand the flow pattern for these modes?)

B) Trapped Vortices (3 pages + plot, 10pts) This problem is based on #5.12 in Acheson. Apply the Helmholtz rule for vortex line motion to obtain the coupled ODEs for the complex-valued positions $z_1(t)$ and $z_2(t)$. Verify the given solution, then plot the implied steady streamfunction and see if it really does resemble Figure 5.19b. (Although it is a bit disturbing that the parenthetical remark at the end Acheson's question is almost contradicted by the question that follows it in the text.)

C) Ring of Vortices (2 pages, 10pts) Solve the problem as posed by #5.14 in Acheson. Presentation and design of notation will be part of the grading. Can you imagine an instance in which this flow pattern would be even remotely related?

(Do you think the configuration is stable?)