Homework #5 • MATH 462 • More Potential Flow

- submit your write-up noon, Friday 05 March.
- in-class midterm reminder: Friday 12 March.
- acknowledge collaborations & assistance.

***) Memory Sheet** (1/2 page, pts on midterm) For reference during the midterm, Acheson appendices A1-A6 will be attached to the exam. You will also be allowed to prepare a memory sheet of formulas & ideas on a 1/2-page of regular notebook paper (single-side only). Memory sheets are to be submitted with the midterm, and will be given credit under the following guidelines:

- no microfilm (reasonable size writing please), 1/2-page single-sided.
- no derivations, only basic formulas & ideas.

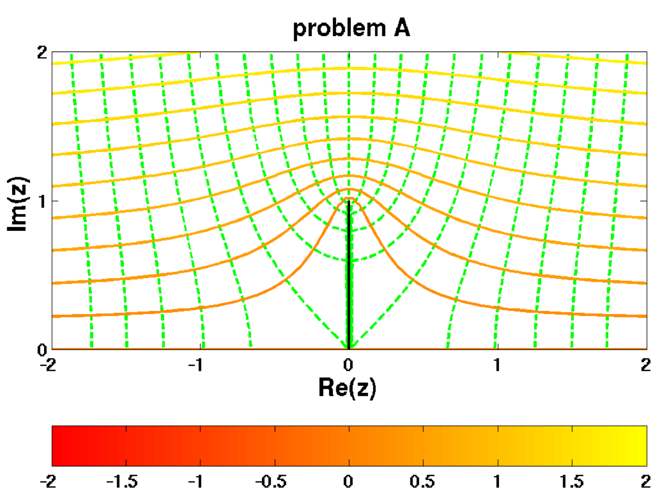
A) Flow past the Wall (2 pages, 10pts) Consider a potential flow as defined by the conformal map $w = \Phi(z)$, where w $= \phi + i\psi$ and $\Phi(z) = \sqrt{z^2+1}$. The streamlines in the z-plane are are curves of constant imaginary w. Consider only the mapping of the upper half z-plane to the upper half w-plane — this also uniquely defines the branch of the square root.

i) Clearly state the branch choice of the square root (i.e. specify how to uniquely evaluate the square root). Then, determine the pre-image of the Re(w)-axis.

ii) Find the equation of the streamline curve in the z-plane which is the pre-image of the coordinate line identified by Im(w) $= \psi$ in the w-plane. Express as an equation in the form $F(x, y; \psi) = 0$.

iii) Then, find an equation of the curve in the z-plane which is the pre-image of the coordinate line identified by Re(w) $= \phi$ in the w-plane. Express as an equation in the form $G(x, y; \phi) = 0$.

iv) Show by an explicit calculation that these two pre-image curves are orthogonal in the z-plane.



B) Outflow (3 pages, 10pts) Consider the potential flow as defined by the complex potential

$$\Phi(z) = Uz + \frac{Q}{2\pi} \ln z$$

for $z \neq 0$. This flow can be plotted for $U = 1, Q = 2$ using *hw05.m*. Calculate the volume flux (per unit height in z) emanating from the origin in three different ways. Take care when the branch cut is involved.

i) by an integration involving the radial velocity on circles $r = a$,

ii) by an integration of the complex potential on arbitrary closed contours enclosing the origin exactly once, and

iii) by an explicit identification of the separating streamlines (first, express the streamlines using the polar form $z = re^{i\theta}$).

Calculate the limiting gap (Δy) between the separating streamlines as $\mathrm{Re}(z) \to \infty$ in two different ways:

iv) by the streamline expression from **iii)**, and

v) by a deduction involving the volume flux.