Homework #01 • MATH495/STAT490 • Introduction to Probability Models

- submit your write-up Wednesday 17 September.
- participation in webct discussions is encouraged.
- please respect page limits & remember to *Guidelines for Reports*.

A) Warm-ups from Ross (2.5 pages, 15 pts) Submit write-ups for problems # 1.10, 1.22 and 1.28 from the first chapter of the text. Please delineate the different problems clearly and highlight your final results & conclusions. (Hint for # 1.22: What is the situation after 2 points are scored?) (Extra hint: one of these solutions is nearly as short as its question.)

general hint: For all probability calculations, identify all the probability concepts: outcomes, sample space, events & known probabilities (especially conditionals).

For further experience in thinking probabilistically, I recommend trying some of the other problems in Ross. Remember that the starred problems have some discussions at the back of the book which are illustrations of different strategies for calculating probabilities. In particular, other problems which I liked were: # 20, 24, 26/27 & 33.

B) Pick 5 Cards ... (1.5 pages, 10 pts) Five cards are chosen (with replacement) from a standard deck of cards. What is the probability that at least one spade was chosen? Calculate this probability in two different ways.

(i) Think negative. Base a calculation on the event:

$$E_N = \{\text{a card is chosen and it is } \underline{\text{not}} \text{ a spade}\} \ .$$

(ii) Think combinatorially. Base a calculation on the events:

$$E_j = \{\text{the } j^{th} \text{ card chosen is a spade}\} \ .$$

Explain how to generalize the probability expression (Ross, p5)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

to the union of more sets. Use this idea to recalculate the probability.

C) Two More (2 pages, 10 pts)

(i) You are told the midterm exam will consist of 6 questions which are to be randomly chosen from a set of 12. One way to use your study time (not recommended) might be to memorize completely 8 of the problems, and know absolutely nothing about the remaining 4. What is the chance you would get 4 or more questions right on the exam?

(ii) Out of N coins, one is double-headed, the rest are ordinary. You choose one coin at random, toss it K times and record all K heads (but oddly enough don't check if it has a tails). What is the probability the chosen coin is double-headed?

D) Probability of Probability (2 pages, 10 pts) We expect that the probability of obtaining any pair in a roll of two dice is 1/6 (why?). Let's test this based upon Bernoulli's definition of probability based on repeated observation:

$$P = \lim_{N \to \infty} \frac{M(N)}{N}$$

where $M(N)$ is the number of occurrences in N realizations. This week's matlab script *w02pairs.m* is a simulation of dice rolling which calculates an empirical (observed) probability based on 100 sets of $N = 36$ dice rolls. Although not the best graphic to use, the script generates a histogram of the empirical probabilities. It also outputs the mean and variance of the 100 sample probabilities.

Rerun the script for increasing N – make a log-log plot of the variance as a function of N. Explain the meaning of this variance result & how it supports Bernoulli's original idea. In particular, how does the probability of observing the probability 1/6 change as you increase N (and also the number of samples, via the variable *Ndata*).