Homework #07 • MATH495/STAT490 • Generating Functions

- submit your write-up before 12 noon on **Thursday** 06 November.
- note page limits, highlight major results.
- **please indicate partners in collaborative efforts. Thank you.**
- to aid the grader, please begin each lettered problem on a new page.

A) Geometric Random Variables (3 pages, 10 pts) The geometric distribution is defined on page 32 of Ross:

$$\text{Prob}\{X = n | n = 1, 2, 3, \ldots\} = P_n = pq^{n-1} \qquad \text{where } q = (1-p) \ .$$

- if X is a geometric random variable, what are the expected values, $\text{E}[(1/2)^X]$ and $\text{E}[z^X]$?
- if X and Y are independent and identically distributed geometric random variables, what is the expected value of $\text{E}[z^{X+Y}] = \text{E}[z^X z^Y]$?
- based on the above result, what is the probability that the sum of two independent, identically distributed geometric random variables is S? Why is this problem hardly different for the sum of n such random variables?

B) Poisson Random Variables (3 pages + 1 page plot, 10 pts) This problem investigates the use of generating functions for Poisson random variables with common mean λ.

- give the generating function for a Poisson distributed random variable;
- if Y is Poisson distributed, give the generating function for the random variable $-Y$ (please don't get this part incorrect);
- using the logic of problem **A)**, use a generating function argument to calculate the mean and variance of the difference $D = X - Y$ of two Poisson random variables;
- verify the above result by simulation & produce an empirical CDF.

- **bonus:** is evaluating the probability distribution for D a reasonable question? If so, do so; if not why not?

C) Raining Again (2 pages, 10 pts) Problems #2 and #3 from Chapter 4 of Ross. The **P** matrix has entries p_{jk} (the element in row j and column k) which are the probabilities of transitioning to state k from state j.