Homework #09 • MATH495/STAT490 • More Markov

- submit your write-up before 12 noon on **Thursday** 20 November.
- please indicate partners in collaborative efforts. Thank you.

***) Snakes & Ladders** (0 pages, 0 pts) In ancient times, before video games, kids would often pass the time playing no-batteries-needed board games. One of these is known as *snakes & ladders* or *chutes & ladders*. An online version of the game is accessible from the class webpage (http://www.snakes-and-ladders.com). Your mission **before monday's lecture** is to win the game twice, and carefully count how many dice rolls it took you to reach square #100. (Note: each roll of six counts as a separate roll.) Write your roll counts on the board when you enter the classroom for lecture. Bring a calculator to class.

1	3	4
2	5	8
6	7	9

A) Simple Maze, Part I (4 pages max, 10 pts) A lab rat is put into a simple maze made up of a 3×3 array of cells, where each cell is connected to its neighbour. This is shown in the figure above where the cell numbering scheme is shown. For instance, from cell #1 you can reach only cells #2 and 3; whereas the central cell #5 connects with cells #2, 3, 7, 8. Assume that our rat (named Markov P) occupies one cell of the maze and moves to adjacent cells by a random choice that is equally likely among all possible neighbouring cells.

- explain why our rat's behaviour is a Markov process (how many states?);
- find the unique stationary probability vector (the answer has a nice interpretation, can you see it?);
- if a snack is put in cell #9 and our rat starts in cell #1, use the transience time matrix $\mathbf{S} = [s_{ij}]$ to calculate the expected time until Markov P first finds the food. (Hint: make cell #9 an absorbing state.)

B) Simple Maze, Part II (4 pages max, 10 pts) Finally, if you are now allowed to close exactly two of the connections in the maze, (before discussing with others) design two new mazes — one which increases Markov P's expected time to snacking, the other which decreases the time. Explain the rationale behind your choices, and then adapt your calculation above for your new mazes. You may then discuss with others on webct to see what the optimal mazes might be.

C) First Return Time (3 pages, 10 pts) Consider a maze which is a closed ring of six cells. Markov P moves clockwise with probability p and counterclockwise with probability $q = 1 - p$. Identify one of the cells as the 0 cell and number the others 1 through 5 in clockwise order. Define r_i = expected number of transitions to first enter state 0 from state $i = 1 \ldots 5$. Conditioning on the first transition, write 5 equations for the r_i. Use these results to find, or plot, the mean first return time (starting from state 0, ending at state 0) as a function of p. (You should use some computational assistance.)