<u>Fall 2001: Math 901, Advanced Mathematical Methods</u>

Beyond Separation of Variables: Exact & Approximate Techniques for Solving PDEs

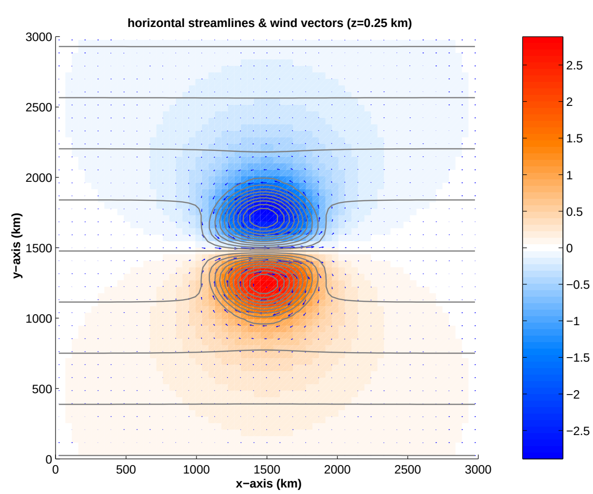
The simplest approach to solving PDEs (partial differential equations) is the method of separation of variables. The course discussion will begin from a natural extension of separation that leads to integral solution techniques, which includes the Fourier and Laplace transform. Investigation of this solution perspective will establish the close connection between complex variable theory and differential equations. Another class of exact integral techniques are based upon the convolution or Greens function methodology. These integral-based approaches also offer an elementary entry into the realm of special functions.

In many cases, the understanding and interpretation of exact solutions requires numerical computation and/or further approximation. The study of asymptotic and perturbation methods provide a systematic framework for the approximate analysis of PDE solutions. Here, the presentation will begin from the far-field approximation of integral solutions, and continue onto developments in multiple-scale, averaging and boundary-layer methods.

Lectures will be based upon a *case-study* approach of PDE examples. Computational illustration will be an important tool for the lectures and assigned work. The rudiments of numerical computing will be developed through the use and modification of downloadable Matlab scripts.

Background familiarity: elementary differential equations (ODEs & PDEs), complex variables & Matlab computing.

Further information & updates: www.math.sfu.ca/∼muraki



The image above shows the streamlines and wind vectors for a surface dipole from a model of atmospheric flow. The dipole is obtained as a solution to a three-dimensional Laplace PDE with mixed boundary conditions. Its computation involves a numerical implementation of an integral Hankel transform – an example of applied special function theory.