Investigation #5 • APMA 900 • Boundary-Layer Theory

- submit your write-up by Wednesday 28 November.

A) Right-Side Boundary-Layer (4 pages): Rework the example from lecture

$$-\epsilon\, u_{xx} + u_x = f(x) \qquad ; \qquad u(-1) = u(1) = 0$$

for $\epsilon \to 0^+$ and where the sign of the second derivative term is taken to be negative. Extend the solution representation to one which has an $O(\epsilon^6)$ error that is uniform over the whole interval $-1 \le x \le +1$. Present a graphic which clearly demonstrates this uniformity of the error.

B) Non-Constant Coefficient (4 pages) The following is based upon Problem 9.20 in Bender & Orszag. For what values of the real parameter α does the solution to the ODE boundary value problem

$$\epsilon\, y_{xx} + x^\alpha y_x + y = 0 \qquad ; \qquad y(0) = y(1) = 1$$

have a boundary-layer at $x = 0$? It is also necessary to demonstrate that the asymptotic matching of the leading order is possible. (Hint: careful attention to detail will pay off.) The boundary scaling will need an approach similar to that used in Section 2.2 of Holmes.

Bonus: The webpage has an ODE solver for this problem that is not very well-behaved for small ϵ and seeming good values of α. Improve its performance.