Investigations #01 • APMA 935 • Propagation

- submit report at my office/mailbox, by Noon Friday 20 January.
- the goals for your written reports are outlined in the *Guideline* (on the webpage).
- discussions are encouraged, collaborations are to be acknowledged, but written reports are to be individually prepared.
- the spirit of the page limits are to be respected. additional efforts may be included as attached (but clearly delineated) appendices.

***)** The intention of parts **A)** and **C)** is not to be a coding/debugging intensive project. In this spirit, I'm requesting that a common set of numerical core(s) (just the PDE solution routine, not the I/O) be posted on the webct discussions page. These pieces of matlab script should have a header which documents the evolution & acknowledges the contributions of updates. Thank you.

(Note: I also suspect now that Wilson's contribution may also be second-order. If so, it's a subtle issue, since the continuity of u_x will have been tacitly imposed in the scheme. That is, if this BC is changed, it is not clear how the scheme needs to be altered.)

A) Second-order Accuracy (2 pages incl. plots) Verify that an implementation of one of the finite-difference schemes from Carrier/Pearson can be adapted to give a second-order accurate simulation of the solution to the interface scattering problem:

$$u(x,t) = \begin{cases} g_I(t - x/c_-) \quad + \quad A_R\, g_I(t + x/c_-) & \text{for } x < 0 \\ A_T\, g_I(t - x/c_+) & \text{for } x > 0. \end{cases} \tag{1}$$

You may use the posted explicit scheme from the lecture (*w01wave.m*) — although I've not checked its accuracy. I also think it would be great if interested parties could modify it to benchmark one of the implicit schemes.

B) Conservation of Energy (2 pages) Clarify the derivation of energy for the case of a discontinuous wavespeed with the BCs from the lecture. For an incoming wave $g_I(t - x/c)$ with compact support, show analytically that the energy is conserved in (1) before and after the interaction with the interface.

C) Impedance Matching (4 pages incl. plots) In lecture, it was shown that, for an incoming mono-chromatic (pure harmonic) wave, perfect impedance matching can be achieved, i.e. zero reflection. In practice, the waves are only nearly mono-chromatic (except perhaps in power-transport applications), as in cable TV transmissions which are signals containing a narrow bandwidth of frequencies.

Based on time-dependent simulations, design a quantitative demonstration that, for amplitude-modulated harmonic signals, the impedance matching remains essentially optimal for the parameter values (c_2 and L) derived for the mono-chromatic coupler.

Discussion of strategies for the computational design are definitely encouraged.