

Investigations #02 • APMA 935 • A WKB Exercise

- Final write-up due, by Noon Friday 03 February. Please submit a progress report to webct by Sunday 29 January.

- A) Eigenvalues** (3 pages) Apply the WKB approach presented in lectures to the problem of finding highly-oscillatory solutions to the wave equation

$$u_{tt} - c^2(x) u_{xx} = 0 \tag{1}$$

where the wavespeed is  $2\pi$ -periodic. For time-harmonic solutions, the additional imposition that the waves be  $2\pi$ -periodic in space yields an eigenvalue problem for the frequency  $\omega$ . Derive an asymptotic expression for large values of these frequencies. Carry out the expansion to include the first correction to the frequency.

**Group Challenge:** Verify the eigenvalue expression numerically.

**Bonus:** I was not able to devise a particular example of  $c(x)$  for which the integrals could be evaluated explicitly. Is there in fact a reasonable class of examples?

- B) Reflection Coefficient** (4 pages) Apply the methodology from lecture to calculate the high-frequency asymptotic behaviour of the reflection coefficient for the wavespeed profile

$$c(x) = \frac{c_0}{1 + a \operatorname{sech}^2 2x} \tag{2}$$

for  $-1 < a < 0$ .