Investigations #05 • APMA 935 • Branch Analysis & Water Waves

- Final write-up due, by Noon Friday 17 March. Please submit a progress report to webct by Monday 13 February.

A) **Water Waves** (4 pages + extra) Chapter 4 of the text presents the derivation of the linear theory for surface water waves. One of the key results of section 4.2 is the linear dispersion relation for one-dimensional fourier mode

$$\omega(k) = \sqrt{gk \tanh kh}$$

where h is the undisturbed water depth. For the problem of an initially depressed surface

$$\eta(x, 0) = -e^{-x^2} \qquad ; \qquad \phi(x, y, t) = 0$$

compute the dynamics of the spreading disturbance. Then apply a stationary phase analysis to explain the evolution of the ring pattern.

extra: The text also gives energy density formulas for the water wave system – investigate (numerically and, perhaps asymptotically?) the idea that integrated energy between group rays is conserved.

B) **A Branch Analysis Problem** (6 pages) Investigate the Green's function for the damped wave equation

$$u_{tt} + 2\epsilon u_t - u_{xx} = 0$$

for the choice with the derivative being a delta function, $u_t(x, 0) = \delta(x)$. The Fourier transform approach will have considerable similarity with the case done in class.

Once your branch analysis is complete, you may either: **(i)** find an integral representation which will allow you to compute some profiles $u(x, t)$, or **(ii)** find a special function representation of the solution.