APMA 935 • Spring 2006

Mathematical Models of Wave Propagation

In the physical world, the propagation of waves represents one of the fundamental mechanisms for the transport of energy. Historically, the early development of the mathematics of waves followed scientific advances in optics, elasticity, hydraulics and acoustics. Modern wave theory finds everyday use in applications such as laser technology, earthquake detection, climate prediction and ultrasound imaging. The central theme of this class will be the analysis and computation of wave behaviour in partial differential equations that arise in models of physical wave processes.

Beyond the familiar hyperbolic wave equation and its simple solutions of translation, a rich theory of linear wave propagation emerges from the effects of dispersion and spatial dimension. In dispersive systems, the concepts of phase and group velocity are fundamental for understanding the propagation characteristics of waves. The theory of rays and Huygens' principle of wavefronts are naturally obtained as the behaviour of waves in the limit of high frequency. These ideas provide the basic mathematical tools for investigating models for common wave phenomena:

- reflection, refraction & focussing,
- diffraction, interference & scattering,
- waveguides & resonance.

Nonlinearity adds further avenues for complexity — solitary waves, instabilities and shocks are just a few examples which are described through elementary nonlinear wave equations.

Computer visualization will be an important accompaniment to the lectures and assigned work. Methods for numerical computing and graphics will be introduced through the use and modification of downloadable Matlab scripts.

Readings: Wave Motion, J. Billingham & A.C. King, Cambridge Univ. Press (2000)

Further information & updates: www.math.sfu.ca/∼muraki

The above panels represent a wave depiction of the Greek classical elements: fire, earth, air and water. An expanding blast wave (bright ring) of debris emanates from Supernova 1987A. Furrows of sand waves corrugate a desert scene. A wave cloud "breaks" high above New Zealand. Swimming ducks leave a wake of dispersing waves.