Investigation #2 • Models of Nonlinearity (math 990) • Variations & Modelling

- e-mail progress due Friday 16 February (send to *math-990*).
- final report ($\sim$5 pages) due Wednesday 21 February.

A) Investigate the effects dimensionality on the bifurcations induced by diffusion and nonlinearity

$$u_t = \nabla^2 u + A\,u - B\,u^3$$

where ∇^2 denotes the appropriately dimensioned Laplacian. Consider BCs that are zero on $0 \leq x \leq L_x$, $0 \leq y \leq L_y \ldots$ For instance, the small amplitude perturbative expansion will give quantitative results. The aspect ratios of the domain will also affect the bifurcation. Your report should address why the results make intuitive sense.

B) The variational analysis shows that the steady-states $\bar{u}(x)$ of

$$u_t = \nabla^2 u + A\,u - B\,u^3 \qquad ; \qquad u(0,t) = u(\pi,t) = 0$$

are minima of the functional

$$I[u] = \int_0^\pi \left\{ \frac{1}{2}u_x^2 - \frac{A}{2}u^2 + \frac{B}{4}u^4 \right\} dx \ .$$

Discuss the results when one substitutes a truncated Fourier series

$$\bar{u}(x) = a_1 \sin x + a_2 \sin 2x + a_3 \sin 3x + \ldots$$

and minimizes the functional over the coefficients.

C) Extend the monopole model for the magnetic rotor toy to incorporate the magnetic *dipole* effect. (Note, dipole here is used in the magnetostatic sense, not in the mathematical sense whereby a dipole is the zero separation limit of two oppositely signed monopoles.) Be quantitative in presenting your modelling results.

D) Develop a quantitative and systematic study using the computational monopole model. One idea is to demonstrate the existence of chaotic trajectories by computing either: (a) Poincaré maps following the discussion in Drazin (s8.3), or (b) Lyapunov exponents (see me for computing references). Another idea is to correlate systematically the trajectory behaviours with the (Hamiltonian) energy.