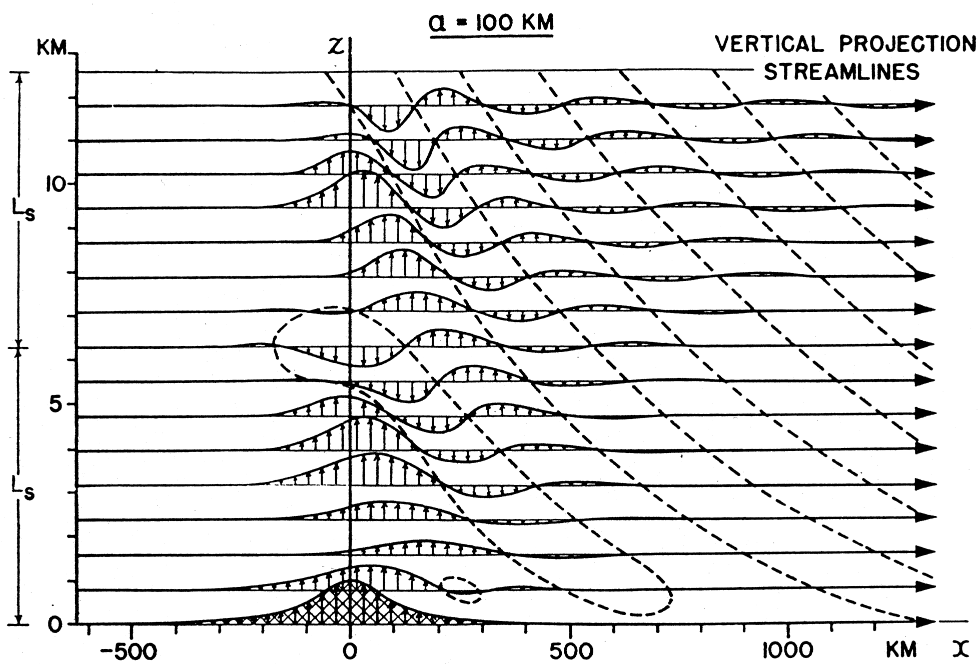
Gravity Waves with Topography . . .

. . . and Possibly Without?

stratified, hydrostatic & rotating flow
balance, waves & applied mathematics



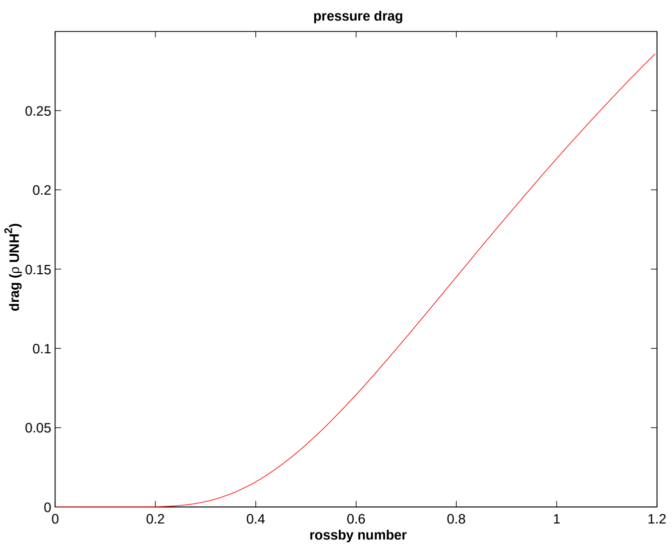
Queney, 1947

▷ Chris, Rich, Joe, Craig

Flow Over Topography

- QG theory: cap vortex only
- Queney theory: $\mathcal{R}$-transition from QG to waves
- pressure drag as indicator of wave strength (2D ridge)

$$\text{drag} \sim \frac{1}{\mathcal{R}}\, e^{-1/\mathcal{R}}$$



- transition-like exponential increase in wave action

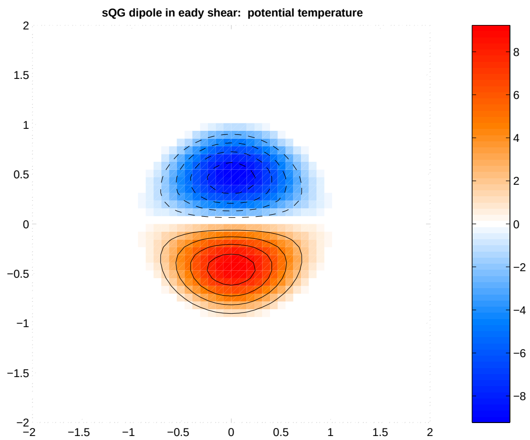
Wave Generation

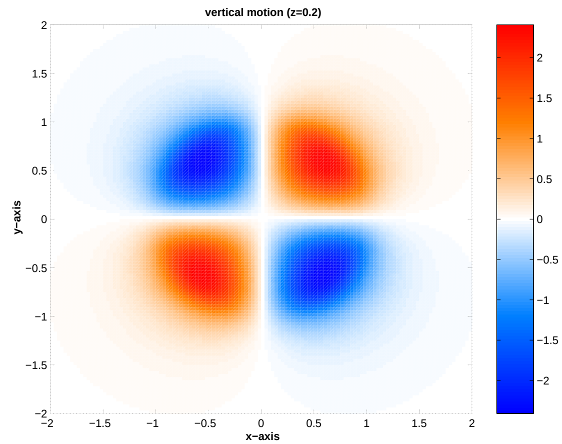
A General Mechanism

- ▷ lessons from topography:

  w ind relative to vertical motion $\rightarrow$ waves

  w aves are exponentially small when $\mathcal{R}$ small





Wave Generation within <u>Balanced</u> Flows?

- ▷ vertical motions from QG: <u>omega equation</u>

  Q G is zero Rossby number limit

- ▷ IDEA: wave generation as finite-$\mathcal{R}$ correction to QG flow?
- ▷ experiment: gravity wave wake from a QG dipole

Quasigeostrophy

Rotating, Stratified Flow

- zero Rossby number limit
- geostrophy

$$\begin{aligned} v &= \phi_x \\ -u &= \phi_y \\ \theta &= \phi_z \end{aligned}$$

- representation of all variables by potentials
- PV dynamics & inversion

$$\frac{Dq}{Dt} = 0 \qquad ; \qquad \nabla^2 \phi = q \qquad \text{(with BCs)}$$

- vertical motion from $\vec{Q}$-vector

$$\nabla^2 w = \nabla \cdot \vec{Q} \qquad \text{(with BCs)}$$

- waves purged by construction

Beyond Quasigeostrophy

Three-Potential Representation

- finite Rossby number
- wind & vertical motion

$$\begin{pmatrix} u \\ v \\ \mathcal{R}w \end{pmatrix} = -\nabla \times \begin{pmatrix} G \\ -F \\ \Phi \end{pmatrix} = \begin{pmatrix} -\Phi_y & -F_z & \\ +\Phi_x & & -G_z \\ & +F_x & +G_y \end{pmatrix}$$

- potential temperature

$$\theta = \nabla \cdot \begin{pmatrix} G \\ -F \\ \Phi \end{pmatrix} = G_x - F_y + \Phi_z$$

- PV dynamics

$$\frac{Dq}{Dt} = 0$$

- three inversions

$$\begin{aligned}
\nabla^2 \Phi &= q + \mathcal{R} \left(|\nabla \Phi_z|^2 - (\nabla^2 \Phi)\, \Phi_{zz} \right) + \ldots \\
\nabla^2 F &= + \mathcal{R} \left(- \left(\frac{D\theta}{Dt} \right)_x + \left(\frac{Dv}{Dt} \right)_z \right) \\
\nabla^2 G &= + \mathcal{R} \left(- \left(\frac{D\theta}{Dt} \right)_y - \left(\frac{Dy}{Dt} \right)_z \right)
\end{aligned}$$

F, G Correction Potentials

- finite Rossby number
- three inversions

$$\begin{aligned}\nabla^2 F &+ \mathcal{R}\,(G_{xx} - F_{xy} + G_{zz})_t &= Q^x &= 2\,\mathcal{R}\,J(\Phi_z, \Phi_x)\\ \nabla^2 G &+ \mathcal{R}\,(G_{xy} - F_{yy} - F_{zz})_t &= Q^y &= 2\,\mathcal{R}\,J(\Phi_z, \Phi_y)\end{aligned}$$

- surface BCs:

$$F_x + G_y = \mathcal{R}\,w \qquad\qquad G_x - F_y = 0$$