

ON THE INTERSECTION OF INFINITE GEOMETRIC AND ARITHMETIC PROGRESSIONS

Abstract: In this talk, we present results from the joint paper by Artūras Dubickas and Jonas Jankauskas, which is to appear in *Bulletin of the Brazilian Mathematical Society*. We prove that the intersection $\mathcal{G} \cap \mathcal{A}$ of an infinite geometric progression $\mathcal{G} = u, uq, uq^2, uq^3, \dots$, where $u > 0$ and $q > 1$ are real numbers, and an infinite arithmetic progression $\mathcal{A}$ contains at most 3 elements except for two kinds of ratios q . The first exception occurs for $q = r^{1/d}$, where $r > 1$ is a rational number and $d \in \mathbb{N}$. Then this intersection can be of any cardinality $s \in \mathbb{N}$ or infinite. The other (possible) exception may occur for $q = \beta^{1/d}$, where $\beta > 1$ is a real cubic algebraic number with two nonreal conjugates of moduli distinct from β and $d \in \mathbb{N}$. In this (cubic) case, we prove that the intersection $\mathcal{G} \cap \mathcal{A}$ contains at most 6 elements.

We shall discuss how this problem can be transformed in to the language of fractional parts of powers of algebraic numbers and how it is related to multiplicity of rational linear recurrent sequences. We will give the sketch of the proof, which is based on the geometry of complex zeros of trinomial equations.