<u>MATH 470:</u>
<u>Calculus of Variations</u>

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Recall that an important application of calculus is to optimize (maximize or minimize) a given function of one or more variables.
In the <u>*Calculus of Variations*</u>, we study a generalization of this problem: Find the unknown *functions* which optimize quantities expressed in terms of *integrals*.

Example — the prototypical "brachistochrone" problem: Find the shape of a frictionless wire down which a particle slides between two given points in the shortest time.
While they have long been particularly useful in mechanics and other areas of physics, variational methods are now productively applied in fields as diverse as geometry, economics, engineering and finance.

An application — *Optimal Control:* In systems described using differential equations, one seeks functions minimizing integrals representing a "cost".
For instance, an example of a biological control problem: Find the optimal dosage (over time) of an anti-cancer drug to reduce the number of tumour cells to a prescribed level, while minimizing the cumulative toxicity of the drug.

In this introduction to the calculus of variations and optimal control, we will encounter various applications while carefully developing much of the relevant mathematical theory.

<u>Prerequisites:</u> *The necessary background is mainly a strong understanding of multivariable calculus and ordinary differential equations. However, students will be assumed to have the mathematical and modelling experience ("maturity") appropriate to a 4th-year applied mathematics course, which is best obtained by completion of one or more of the prerequisite courses.*
(See the calendar for formal prerequisites. Questions? Contact the lecturer.
Interested graduate students should register for MATH 770.)

<u>Textbook:</u> Mike Mesterton-Gibbons, *A Primer on the Calculus of Variations and Optimal Control Theory*, American Mathematical Society (2009).