MATH 252-3 **Spring 2004**

Vector Calculus

Midterm Exam: Old Exam Our Exam Friday, 20 February 2004

Course Web Site: `http://www.math.sfu.ca/~ralfw/math252/`

The following is a Math 252 test from a previous year; this year, we have not yet covered the material for question 3, nor for question 4 (though you should be able to do this question based on what we have done). The total number of marks is 60.

1. (10) Prove, using tensor notation, the following vector identity:

$$\nabla \times (\mathbf{F} \times \mathbf{G}) = (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{F} + (\nabla \cdot \mathbf{G})\mathbf{F} - (\nabla \cdot \mathbf{F})\mathbf{G}.$$

2. (3) Evaluate the following:

$$\nabla \times \nabla \left(\frac{x^3 y\sqrt{z} + e^{xy}\cos(yz) - \tanh^{-1}\left(\frac{x}{y}\right)}{\ln(x^2+z^2) + 3xz^{4/3}y^{\pi\cos(x)}} \right) \quad .$$

3. (5) What is $\nabla\mathbf{F}$ when written as a dyadic?

4. (10) Let $f(x,y,z) = \sin(px)\sinh(qy)e^{rz}$ where p, q and r are constant. What are the necessary conditions on p, q and r to make $f(x,y,z)$ satisfy Laplace's equation?

5. Let $\mathbf{F} = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$.

   (a) (2) What is $\nabla \cdot \mathbf{F}$?

   (b) (3) What is $\nabla \times \mathbf{F}$?

   (c) (5) What are the equations for the flow lines of $\mathbf{F}$ going through the point (x_0, y_0, z_0)?

6. (10) Find the equation of a plane tangent to the surface $z = x^2 + y^2$ at the point $(3, 4, 25)$.

7. Consider the space curve defined by the following:

$$x = e^t \cos t, \quad y = e^t \sin t, \quad z = 0$$

   and assume there is a particle moving along this curve as a function of time t.

   (a) (2) What is the speed?

   (b) (2) What is the tangential component of acceleration?

   (c) (2) What is the normal component of acceleration?

   (d) (2) What is the unit tangent vector $\mathbf{T}$?

   (e) (2) What is the curvature of the curve?

   (f) (2) What is the torsion of the curve?