MATH 252-3 **Spring 2004**

Vector Calculus

Homework Set 10 Due Friday, 26 March 2004

Course Web Site: `http://www.math.sfu.ca/~ralfw/math252/`

Problems from Davis and Snider "Introduction to Vector Analysis":

- Section 4.5 (pp.222–223): 1, 10 (see note 1 below)
- Section 4.6 (pp.236–237): 2, 4, 5 (plot the surface using Maple), 10 (see note 2)
- Section 4.7 (pp.246–249): 4, 8, 12 (plot the surface using Maple), 18, 20 (see note 3)

Notes:

1. For problem 10 in Section 4.5, compute two different vector potentials: find $\mathbf{G}_1$ using formula (4.18) (integrate along a line segment), and find $\mathbf{G}_2 = \chi\mathbf{k}$ using the method on p.221, where χ is a scalar potential for $\mathbf{k} \times \mathbf{F}$ (this easier method works specifically for two-dimensional vector fields; we didn't manage to cover this in class); in each case, check that $\mathbf{F} = \nabla \times \mathbf{G}$.

2. Problem 10 in Section 4.6 is quite quick starting from

$$dS = \left| \frac{\partial \mathbf{R}}{\partial u} \times \frac{\partial \mathbf{R}}{\partial v} \right| du\, dv \ ,$$

   and using the identity $|\mathbf{A} \times \mathbf{B}|^2 = |\mathbf{A}|^2|\mathbf{B}|^2 - |\mathbf{A} \cdot \mathbf{B}|^2$ (see Section 1.14 problem 7, or the second note on Homework Set 3).

3. For problem 20 in Section 4.7, plot the surface using Maple for $0 \leq v \leq 2\pi$, and calculate the surface area for this range of v.

Extra problem:

1. *Fun with the Möbius Strip...*
   Construct a Möbius strip by taking a strip of paper, giving it a single twist and taping the ends together. Convince yourself that the resulting surface has only one side.

   (a) Cut the Möbius strip down a central line (can you predict the result in advance?), and report your observations. (As suggested by a Math 252 student:) You may also wish to cut a Möbius strip along 1/3 of the width of the strip; what happens?

   (b) What happens if you use a double twist instead of a single twist?