MATH 252-3 **Spring 2004**

Vector Calculus

Homework Set 11 (last set!) Due Tuesday, 6 April 2004

Course Web Site: `http://www.math.sfu.ca/~ralfw/math252/`

Problems from Davis and Snider "Introduction to Vector Analysis":

- Section 4.8 (pp.256–257): 6
- Section 4.9 (pp.262–265): 3(b), 5, 8, 12, 17, 20, 29, 30, 34
- Section 4.10 (pp.271–272): 1, 5
- Section 5.1 (pp.276–277): 6, 7, 8
- Section 5.4 (pp.294–295): 6, 7, 11
- Section 5.5 (pp.299–300): 1, 2

Extra problem:

1. *Vector-valued Integrals*
   Surface and volume integrals of vector-valued functions may be defined as limits of Riemann sums, in a similar way to the definition of integrals of scalar-valued functions; alternatively, one may consider them componentwise; for instance for $\mathbf{G} = G_1\mathbf{i} + G_2\mathbf{j} + G_3\mathbf{k}$, we can define the volume integral as

$$\iiint_V \mathbf{G}\, dV = \mathbf{i} \iiint_V G_1\, dV + \mathbf{j} \iiint_V G_2\, dV + \mathbf{k} \iiint_V G_3\, dV.$$

   We can use the divergence theorem, or the corresponding results for components in the form

$$\iiint_V \frac{\partial G_1}{\partial x}\, dV = \iint_S G_1 \mathbf{i} \cdot \mathbf{n}\, dS \ , \qquad \text{(and similarly for } y,\ z \text{ components)}$$

   to obtain identities for such vector-valued integrals (where S is the surface of the region V, and $\mathbf{n}$ is the outward unit normal).

   (a) Let f be a scalar field. Show the identity

$$\iiint_V \nabla f\, dV = \iint_S f\mathbf{n}\, dS \ \left(= \ \iint_S f\, d\mathbf{S} \right) \ ,$$

   and hence deduce a vector interpretation of the limit

$$\lim_{V \to \mathbf{0}} \frac{\iint_S f\mathbf{n}\, dS}{V} \ .$$

   [You should use two different methods to show the first identity: (a) Establish the result componentwise; (b) Apply the divergence theorem to the vector field $\mathbf{G} = f\mathbf{C}$, where $\mathbf{C}$ is an arbitrary *constant* vector.]

   (b) Let $\mathbf{F}$ be a vector field. Show the identity

$$\iiint_V \nabla \times \mathbf{F}\, dV = \iint_S \mathbf{n} \times \mathbf{F}\, dS,$$

   and hence give a vector interpretation of the limit

$$\lim_{V \to \mathbf{0}} \frac{\iint_S \mathbf{n} \times \mathbf{F}\, dS}{V} \ .$$