MATH 252-3 **Spring 2004**

Vector Calculus

Homework Set 3 Due Wednesday, 28 January 2004

Course Web Site: `http://www.math.sfu.ca/~ralfw/math252/`

Problems from Davis and Snider "Introduction to Vector Analysis":

- Section 1.12 (pp.51–53): 3, 13, 19, 24, 27, 29
- Section 1.13 (pp.57–59): 3, 7, 10, 11, 15, 17
- Section 1.14 (pp.60–61): **2**, **3**, 5, 6, **7**, 9, 10
- Section 1.15 (p.66): 2

For problems **2**, **3** and **7** of section 1.14, please derive the relevant identities *both* (i) using previously derived identities and properties, using the methods of section 1.14, and (ii) using the tensor notation conventions and methods of section 1.15. Also show problem **7** using the geometric definitions of the dot and cross products.

Notes:

- Observe that section 1.13 problem 17 gives the expansion of a vector $\mathbf{V}$ in terms of any three *independent* vectors $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ (they don't need to be orthogonal).
- Recall the Cauchy-Schwarz inequality

$$|\mathbf{A}\cdot\mathbf{B}| \le |\mathbf{A}||\mathbf{B}| \quad \Longrightarrow \quad |\mathbf{A}|^2|\mathbf{B}|^2 - (\ \mathbf{A}\cdot\mathbf{B})^2 \ge 0.$$

  The result of section 1.14 problem 7, $|\mathbf{A}\times\mathbf{B}|^2 = |\mathbf{A}|^2|\mathbf{B}|^2 - (\mathbf{A}\cdot\mathbf{B})^2$, provides a formula for the magnitude of the "error" in the Cauchy-Schwarz inequality. It also shows that the Cauchy-Schwarz inequality is an equality exactly when $\mathbf{A}\times\mathbf{B} = \mathbf{0}$, that is, if $\mathbf{A} = \mathbf{0}$ or $\mathbf{B} = \mathbf{0}$ or $\mathbf{A}$ and $\mathbf{B}$ are parallel.