MATH 252-3 **Spring 2004**

Vector Calculus

Homework Set 5 Due Friday, 20 February 2004

Course Web Site: `http://www.math.sfu.ca/~ralfw/math252/`

Problems from Davis and Snider "Introduction to Vector Analysis":

- Section 3.1 (pp.112–114): 4, 5, 7, 10(a), 12, 17, 21, 23, 32, 34
- Section 3.2 (p.117): 3, 4
- Section 3.3 (pp.124–125): 4, 6, 7, 8
- Section 3.4 (p.132): 4, 7, 9, 11
- Section 3.5 (pp.135–136): 10 (relate your answer to that of Section 3.4 problem 9)
- Section 3.6 (p.140): 3

Extra problem

1. Consider the scalar field $f(x, y, z) = x^2 + y^2 - z^2$.
   (a) Plot in Maple the isotimic surfaces given by $f(x, y, z) = x^2 + y^2 - z^2 = C^2$ for $C = 1$, 2 and 3.
   (b) Compute the gradient field $\mathbf{grad} f$, and plot this gradient field in Maple.
   (c) What are the general equations of the flow lines through this gradient field? Write down the equations for the flow lines through the points (1,1,1), (1,1,2) and (1,1,3), and plot these three flow lines in Maple, together with the isotimic surfaces on the same graph.

Notes

1. Note that in problem 4 of Section 3.3 (p.124) you are computing the divergence of the field

$$\mathbf{F} = \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2 + y^2 + z^2)^{3/2}} = \frac{\mathbf{R}}{R^3} = \frac{1}{R^2}\hat{\mathbf{R}},$$

   which is (up to constants) the vector field corresponding to an inverse square force law, such as the gravitational or electrostatic field. The result you should obtain is $div\, \mathbf{F} = 0$ for $R \neq 0$ i.e. $\mathbf{R} \neq \mathbf{0}$; you can interpret the result as saying, for instance, that the electrostatic field generated by a point charge is divergence-free away from the charge.

2. Please look at the statement and result of problem 12 of Section 3.3 (p.124), which gives the mathematical statement of a property that was mentioned in class: The divergence measures the fractional rate of change of volume (you do not need to hand in this question).

3. Problem 12 of Section 3.5 (p.132; optional) has the result that $\mathbf{curl}(f(R)\mathbf{R}) = \mathbf{0}$ for any differentiable function f. This is clear from the interpretation of the curl: a purely radial vector field has zero circulation around any curve, or (maybe more obviously) a vector field pointing purely in a radial direction cannot cause any paddle wheel to rotate.